Problems 1 to 6 of the Rhind Mathematical Papyrus

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When the Rhind Mathematical Papyrus was received by the British Museum in 1864, it was broken in some places, and portions of the brittle fragments were missing. Subsequently the papyrus roll was unrolled and mounted in two glass frames, the first portion, called the Recto (reading from right to left), having a jagged break because of these missing fragments. The whole papyrus was originally 18 ft. long and a little over a foot in width, the second half, called the Verso, having a blank space about 10 ft. long towards the left-hand end, upon which nothing is written.

Section I of the papyrus [1]* consists of an introduction, followed by the division of the number 2 by all the odd numbers from 3 to 101, 50 separate divisions in all, with the answers expressed as unit fractions except for the quite common use of the special fraction 2/3 [2], for which the scribe had a particular liking, so that he used it whenever it was possible. This takes up about one-third of the Recto.

Section II, which is relatively much smaller—occupying only about nine inches of the papyrus—consists of a short table giving the answers in unit fractions of the divisions of the numbers 1 to 9 by 10, followed by Problems 1 to 6 [1] on the division of certain numbers of loaves of bread equally among ten men, in which problems the scribe uses, for his answers, the values given in his table of reference, and then proves arithmetically that they are correct. (This table and an explanatory version are shown on page 62.)

One observes that as soon as it is possible to introduce the fraction 2/3 into his table, the scribe does so, even though for 7 divided by 10 he could have used the simpler value 1/2+1/5, and for 8 divided by 10, 1/2+1/5+1/10, and for 9 divided by 10, 1/2+1/3+1/15 [3].

The problems on the division of loaves are available to us for examination and discussion as a result of the restoration of the missing fragments of the papyrus, which, while in the possession of the New York Historical Society, were recognized as such by the English archeologist Professor Newberry in 1922, and were brought to the British Museum by Edwin Smith to allow them to be put in their appropriate places on the Recto.

In the text of the R. M. P. itself, no statement occurs giving the title, or describing the purpose, of the table. The scribe clearly regarded these as self-evident. The table occurs as if (for example) a student about to solve a group of problems which would require multiplications and divisions of numbers by 7 (let us say) were to write down first of all the 7 times table, for ready reference in the operations which were to follow.

The scribe now sets down the divisions of 1, 2, 6, 7, 8, and 9 loaves among 10 men,

* Numbers in brackets refer to the notes at the nd of the article.

1 divided by	10 is	1/10			
2	"	" 10 "	1/5		
3	"	" 10 "	1/5	1/10	
4	"	" 10 "	1/3	1/15	
5	"	" 10 "	1/2		
6	"	" 10 "	1/2	1/10	
7	"	" 10 "	2/3	1/30	
8	"	" 10 "	2/3	1/10	1/30
9	"	" 10 "	2/3	1/5	1/30

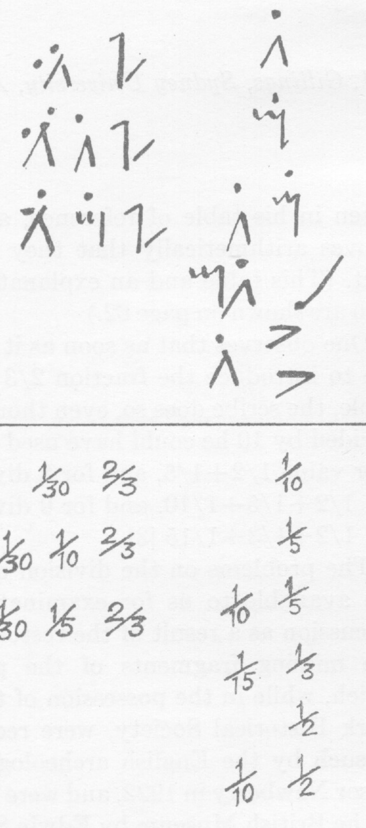


Table of division of 1 to 9 by 10

using the values he has already given in his table. These constitute Problems 1 to 6. He omits the division of 3, 4, and 5 loaves among 10 men, but in this discussion of the Egyptian division of loaves we shall include them, following exactly the methods he uses for the others.

Problem 1

Division of 1 loaf among 10 men.
[Each man receives 1/10.] (From his reference table.)

For proof multiply 1/10 by 10.
Do it thus:

[If]	1 [part is]	1/10
[then]	√ 2 [parts are]	1/5
"	4 "	1/3+1/15

(From the Recto, $2 \div 5 = 1/3 + 1/15$.)

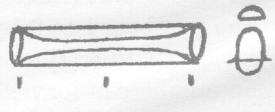
"	√ 8 "	2/3+1/10+1/30

(From the Recto, $2 \div 15 = 1/10 + 30$.)
[Add the fractions on the lines with check marks (since $2 + 8 = 10$): $1/5 + 2/3 + 1/10 + 1/30 = 1$.]
Total 1 loaf which is correct.

Those portions enclosed within square brackets are not given by the scribe, but are included here to explain his methods. He does not prove that 1 divided by 10 is 1/10. His readers are supposed to know this by referring to his table which precedes his problems. Further, the steps in his proof that 1/10 multiplied by 10 is in fact one loaf may be verified by referring to his more elaborate table of "2 divided by all the odd numbers from 3 to 101," with which the R.M.P. begins [4]. Thus, when it is required to multiply the fraction 1/5 by 2, it is only necessary to look in his table, for 2 divided by 5, to find the answer 1/3+1/15. And similarly for 1/15 multiplied by 2, he finds from his table that 2 divided by 15 is 1/10+1/30.

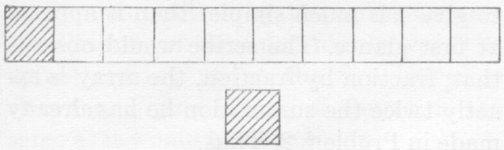
For the addition of the fractions, the standard method, used elsewhere in the R. M. P., was analogous to our modern concept of Least Common Multiple. Taken as parts of (say) the abundant number 30, 1/5 is 6, 2/3 is 20, 1/10 is 3, and 1/30 is 1, so that $6+20+3+1=30$, and thus the whole.

We may fairly assume that the Egyptian loaves were long and regular in cross-section, either cylindrical or rectangular, so that the actual cutting up of a loaf into unit fractions (and of course 2/3) did not present real difficulties. The sign in hieroglyphics for a loaf is



which permits us to make this assumption. The division of one loaf among ten men would be done, therefore, as in Figure 1.

1 *Loaf*



Each man gets 1/10

Figure 1

Problem 2

Division of 2 loaves among 10 men.
[Each man receives 1/5.] (From his reference table.)
For proof multiply 1/5 by 10.
Do it thus:

[If] 1 [part is] 1/5
[then] √ 2 [parts are] 1/3+1/15
(From the Recto, 2÷5=1/3+1/15.)
" 4 " 2/3+1/10+1/30
(From the Recto, 2÷15=1/10+1/30.)
" √ 8 " 1 1/3+1/5+1/15
[Add the fractions on the lines with check marks (since 2+8=10):
1/3+1/15+1 1/3+1/5+1/15=2.]
Total 2 loaves which is correct.

The steps in Problem 2 are practically the same as those for Problem 1, and the division of two loaves among ten men would be done therefore as in Figure 2.

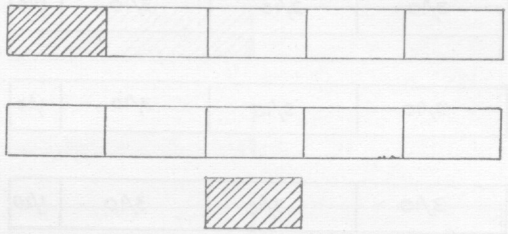
If the scribe had included the division of three loaves among ten men as his next problem, he would have set it down as follows (if we adopt the same procedure shown in the other problems).

Division of 3 loaves among 10 men.
Each man receives 1/5+1/10. (From his reference table.)
For proof multiply 1/5+1/10 by 10.
Do it thus:

If 1 part is 1/5+1/10
then √ 2 parts are 1/3+1/15+1/5
(From the Recto, 2÷5=1/3+1/15.)
then 4 parts are
2/3+1/10+1/30+1/3+1/15
(From the Recto, 2÷15=1/10+1/30.)
then √ 8 parts are
1 1/3+1/5+1/15+2/3+1/10+1/30
[Add the fractions on the lines with the check marks (since 2+8=10):
1/3+1/15+1/5+1 1/3+1/5
+1/15+2/3+1/10+1/30=3.]
Total 3 loaves which is correct.

The addition of the row of fractions is merely (1+2), from the additions in Problems 1 and 2. The division of three

2 *Loaves*



Each man gets 1/5

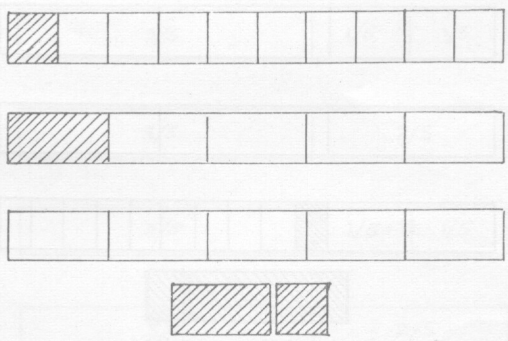
Figure 2

loaves among ten men would therefore be as shown in Figure 3.

It now becomes clear to us why the scribe omitted this division. It is merely a repetition of his first two examples, whose answers were 1/10 and 1/5, giving his answer 1/10+1/5. A glance at Figures 1, 2, and 3 also shows this. Now in the case of the division of one and two loaves equally among ten men, there is no difference whatever between what an Egyptian would have done following the R. M. P. and what would be done in the present day. But in the case of the division of three loaves, a modern division would result in nine men getting 3/10 of a loaf each in one whole piece, and the tenth man getting three smaller pieces each 1/10 of a loaf, as in Figure 4.

The Egyptian foreman charged with the duty of sharing the food fairly and equally among his gang of ten men, would give each man exactly the same share both in quantity (i.e., size) and number of pieces, and thus to the uneducated and ignorant

3 *Loaves*



Each man gets 1/5 1/10

Figure 3

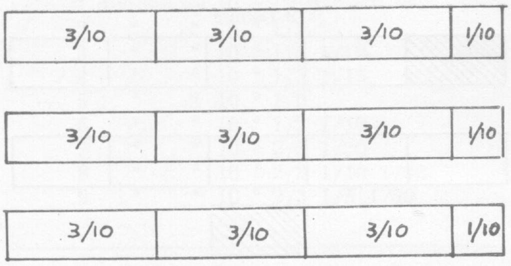


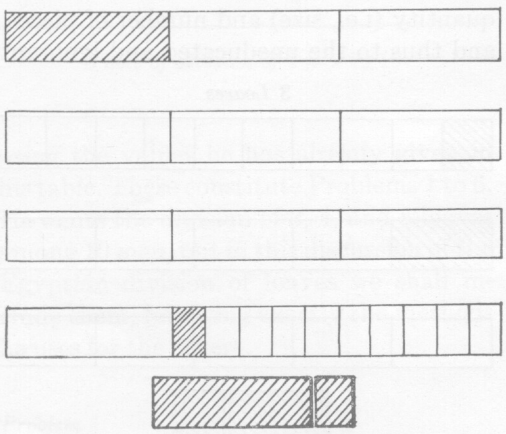
Figure 4

laborers, not only is justice done, but justice also *appears* to have been done. In our modern division it could be supposed that one man had received more than his share, because he had three pieces, to the others' one piece.

We proceed now to the next case, the division of 4 loaves among 10 men. It might have appeared as follows.

Division of 4 loaves among 10 men.
Each man receives 1/3+1/15. (From his reference table.)
For proof multiply 1/3+1/15 by 10.
Do it thus:
  If 1 part is 1/3+1/5
  then √ 2 parts are 2/3+1/10+1/30
(From the Recto, 2÷15=1/10+1/30.)
  then 4 parts are 1 1/3+1/5+1/15
  then √ 8 parts are 2 2/3+1/3+1/15+1/10 +1/30
(From the Recto, 2÷5=1/3+1/15.)
[Add the fractions on the lines with the check marks (since 2+8=10): 2/3+1/10+1/30+2 2/3+1/3 +1/5+1/10+1/30=4.]
  Total 4 loaves which is correct.

4 *Loaves*



Each man gets 1/3 1/15

Figure 5

The addition of the long row of fractions to give 4 is much simpler than it appears at first glance. The scribe would observe that, fraction by fraction, the array is exactly twice the summation he has already made in Problem 2. Thus,

$$2\times 1/3=2/3$$

$$2\times 1/15=1/10+1/30$$

$$2\times 1\ 1/3=2\ 2/3$$

$$2\times 1/5=1/3+1/15$$

$$2\times 1/15=1/10+1/30.$$

The division of four loaves among ten men is therefore as shown in Figure 5.

One again notices that not only is justice done in the Egyptian system, but justice also *appears* to be done. A modern division would result in eight men getting each one portion, namely 2/5 of a loaf, and two men getting each two smaller pieces of 1/5 of a loaf, an apparent inequity perhaps to the ignorant members of the working gang. This is shown in Figure 6.

Continuing we have:

Division of 5 loaves among 10 men.
Each man receives 1/2. (From his reference table.)
For proof multiply 1/2 by 10.
Do it thus.
  If 1 part is 1/2
  then √ 2 parts are 1
  then 4 parts are 2
  then √ 8 parts are 4
[Add the numbers on the lines with check marks (since 2+8=10): 1+4=5.]
  Total 5 loaves which is correct.

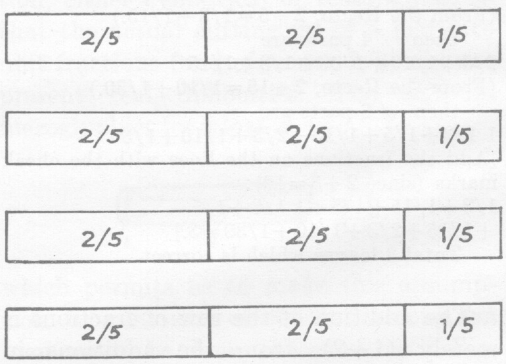


Figure 6

This is the simplest of all the divisions, and it was omitted by the scribe as were the divisions of three and four loaves. The actual cutting up of the five loaves is shown in Figure 7 and, of course, is the same as it would be done today.

Problem 3

Division of 6 loaves among 10 men.
[Each man receives 1/2 + 1/10.] (From his reference table.)
For proof multiply 1/2 + 1/10 by 10.
Do it thus:

[If]	1 [part is]	1/2 + 1/10
[then] √	2 [parts are]	1 + 1/5
"	4 "	2 + 1/3 + 1/15

(From the Recto, 2 ÷ 5 = 1/3 + 1/15.)

" √	8 "	4 + 2/3 + 1/10 + 1/30

(From the Recto, 2 ÷ 15 = 1/10 + 1/30.)
[Add the fractions on the lines with check marks (since 2 + 8 = 10): 1 + 1/5 + 4 + 2/3 + 1/10 1 + 1/30 = 6.]
Total 6 loaves which is correct.

One notes, *en passant*, that the addition of the row of fractions in this problem is, except for the integers, exactly the same as that for Problem 1. The actual cutting up is shown in Figure 8.

This is perhaps the best example to show the efficiency of the Egyptian unit fraction system in the cutting up of loaves, for here again each man gets exactly the same portions, both in quantity and numbers of pieces of bread, whereas a modern division would very likely be as shown in Figure 9.

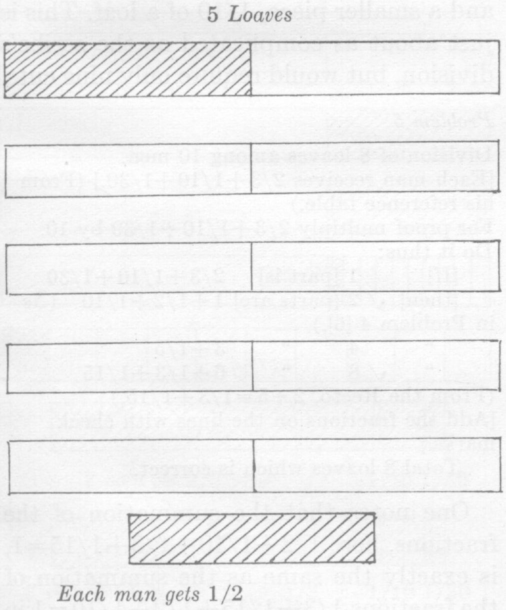


Figure 7

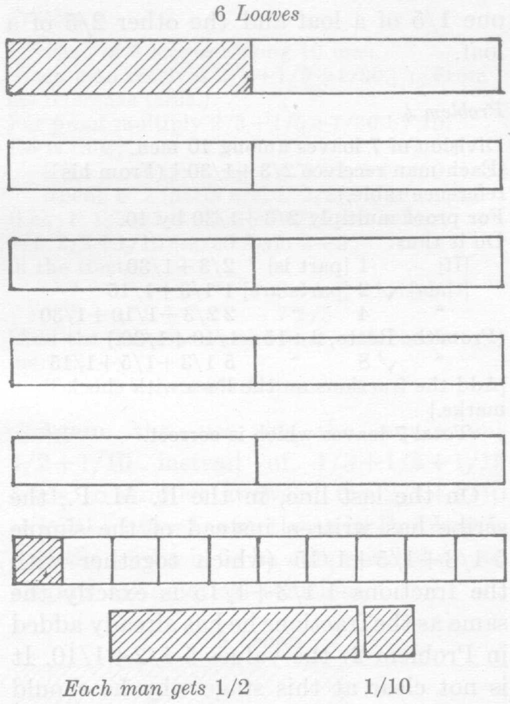


Figure 8

In this case an apparent inequity is even more apparent, since six men each receive a single piece which is 3/5 of a loaf, and four men receive two unequal pieces each,

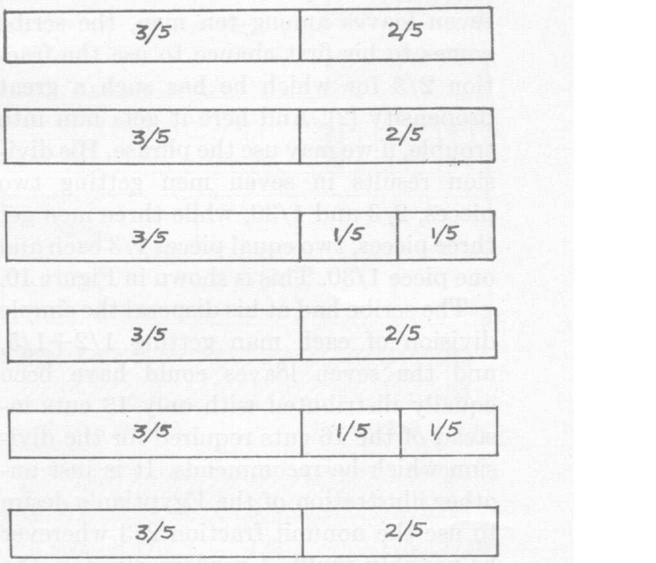


Figure 9

one 1/5 of a loaf and the other 2/5 of a loaf.

Problem 4

Division of 7 loaves among 10 men.
[Each man receives 2/3+1/30.] (From his reference table.)
For proof multiply 2/3+1/30 by 10.
Do it thus:

[If]	1	[part is]	2/3+1/30
[then]	√ 2	[parts are]	1 1/3+1/15
"	4	"	2 2/3+1/10+1/30

(From the Recto, 2÷15=1/10+1/30.)

"	√ 8	"	5 1/3+1/5+1/15

[Add the fractions on the lines with check marks.]
Total 7 loaves which is correct.

On the last line, in the R. M. P., the scribe has written instead of the simple 5 1/3+1/5+1/15 (which together with the fractions 1 1/3+1/15 is exactly the same as the fractions he has already added in Problem 2) the values 5 1/2+1/10. It is not clear at this stage why he should have done this, for while 1/2+1/10 is equivalent to 1/3+1/5+1/15, the substitution does not make the summation to a total of 7 any easier, nor in fact does it make it any more difficult [5]. We shall see later why the scribe preferred to use this alternative form, but the change here made does not in any way alter the general thread of the working of the problem. In the division of seven loaves among ten men, the scribe comes to his first chance to use the fraction 2/3 for which he has such a great propensity [2]. And here it gets him into trouble, if we may use the phrase. His division results in seven men getting two pieces, 2/3 and 1/30, while three men get three pieces, two equal pieces 1/3 each and one piece 1/30. This is shown in Figure 10.

The scribe had at his disposal the simple division of each man getting 1/2+1/5, and the seven loaves could have been equally distributed with only 13 cuts instead of the 16 cuts required for the division which he recommends. It is just another illustration of the Egyptian's desire to use the nonunit fraction 2/3 wherever he possibly could. A modern division, the simplest I can think of, would give seven

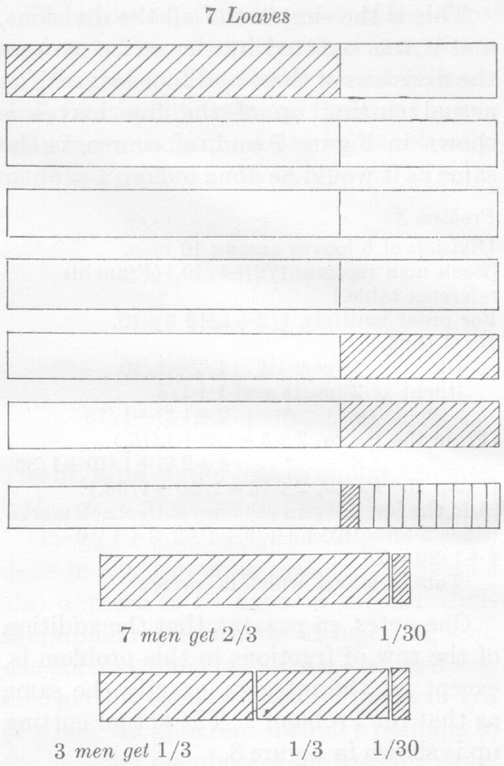


Figure 10

men each one portion of 7/10 of a loaf, and the remaining three men would each receive three pieces, 3/10 of a loaf twice, and a smaller piece, 1/10 of a loaf. This is just about as complicated as the scribe's division, but would require only nine cuts.

Problem 5

Division of 8 loaves among 10 men.
[Each man receives 2/3+1/10+1/30.] (From his reference table.)
For proof multiply 2/3+1/10+1/30 by 10.
Do it thus:

[If]	1	[part is]	2/3+1/10+1/30
[then]	√ 2	[parts are]	1+1/2+1/10 (As in Problem 4 [6].)
"	4	"	3+1/5
"	√ 8	"	6+1/3+1/15

(From the Recto, 2÷5=1/3+1/15.)
[Add the fractions on the lines with check marks.]
Total 8 loaves which is correct.

One notes that the summation of the fractions, here 1/2+1/10+1/3+1/15=1, is exactly the same as the summation of the fractions 1/3+1/15+1/2+1/10=1 in Problem 4, as the scribe wrote it. And now the efficacy of using this alternative form 1/2+1/10 instead of the conventional 1/3+1/5+1/15 becomes quite clear to us, and in the subsequent doubling, 3+1/5 is so much simpler than 2+2/3+1/3+1/15 +1/10+1/30, and the next doubling is simplified as well.

The division of eight loaves among ten men as the scribe directs is shown in Figure 11.

Here seven men get three distinct portions, 2/3, 1/10, 1/30, and three men get four distinct portions, 1/3 and 1/3, 1/10, 1/30, which appear to be more or less identical if the two separate 1/3 pieces are kept together, and so again justice *appears* to have been done. A modern cutting up of loaves would result in eight men getting each 4/5 of a loaf in one piece, while the remaining two men get each four pieces, each 1/5 of a loaf, which might be construed as being inequitable.

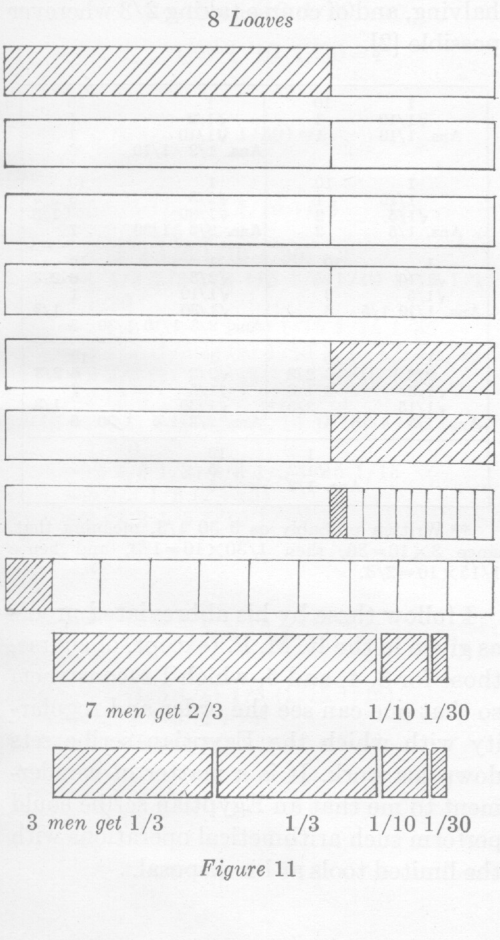


Figure 11

Problem 6

Division of 9 loaves among 10 men.
[Each man receives 2/3+1/5+1/30.] (From his reference table.)
For proof multiply 2/3+1/5+1/30 by 10.
Do it thus:

[If]	1 [part is]	2/3+1/5+1/30
[then]	√ 2 [parts are]	1 2/3+1/10+1/30

(i.e., 1 1/3+1/3+1/15+1/15 as before, =1 2/3+1/10+1/30 from 2÷5 in the Recto.)

"	4	" 3+1/2+1/10 [6]
"	√ 8	" 7 1/5

[Add the fractions on the lines with check marks.]

Total 9 loaves which is correct.

Again the use of the equivalent 1/2+1/10 instead of 1/3+1/5+1/15 leads to simpler answers, for 2/3+1/10 +1/30+1/5=1, exactly as in Problem 1, and the scribe is to be commended on an elegant arithmetical solution.

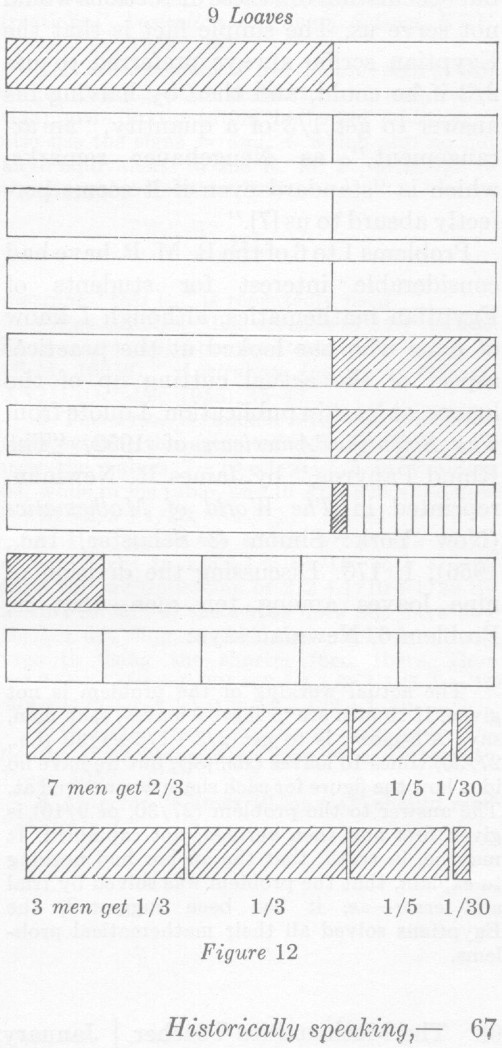


Figure 12

The division would be performed as in Figure 12, and the apparent equity of the shares is observable if the two separate 1/3 pieces are placed together to make 2/3.

A modern distribution of nine loaves among ten men would result either in nine men receiving 9/10 of a loaf and the tenth man getting nine small pieces each 1/10 of a loaf, which would be clearly inequitable to the uneducated laborer, or in the ten men getting each 1/2 a loaf, and then eight of them receiving in addition 2/5 of a loaf while the remaining two men would get four small pieces each 1/10 of a loaf, which is even worse.

As in Problem 4, the scribe had at his disposal simpler divisions in terms of unit fractions, as for example 1/2+1/5+1/10 for 8÷10, and 1/2+1/3+1/15 for 9÷10, but speculations on these dissections would not serve us. The simple fact is that the Egyptian scribe always preferred to use 2/3 if he could, and then by halving his answer to get 1/3 of a quantity, "an arrangement," as Neugebauer remarks, which is "standard even if it seems perfectly absurd to us [7]."

Problems 1 to 6 of the R. M. P. have had considerable interest for students of Egyptian mathematics, although I know of none who has looked at the practical aspect of the actual cutting up of the loaves, at least in publication. I quote from *The Scientific American* of 1952, "The Rhind Papyrus," by James R. Newman, reprinted in *The World of Mathematics* (New York: Simon & Schuster, Inc., 1956), I, 173. Discussing the division of nine loaves among ten men, that is, Problem 6, Newman says:

> The actual working of the problem is not given. If 10 men are to share 9 loaves, each man, says A'h-mosè, is to get 2/3+1/5+1/30 (i.e., 27/30) times 10 loaves (*sic.* [8]); but we have no idea how the figure for each share was arrived at. The answer to the problem (27/30, or 9/10) is given first and then verified, not explained. It may be, in truth, that the author had nothing to explain, that the problem was solved by trial and error—as, it has been suggested, the Egyptians solved all their mathematical problems.

But of course we know how he would have obtained the values he gives in his table! The first part of the Recto of the R. M. P. is filled with examples of how he would do it. Chace (R. M. P., Vol. 1, p. 23), for example, gives his methods for 3 and 7 divided by 10, but they are so simple from the scribe's point of view that he merely sets the answers down in order, without even a heading let alone any explanations. A'h-mosè neither here nor elsewhere indulged in unnecessary detail or verbosity. He was writing a mathematical text! Having "no idea how the figure for each share was arrived at," as Newman puts it, is so far from the truth that I will set down here how the scribe could have performed the divisions of the numbers 1 to 9 by 10. All that is necessary is to note that he performed division by continually multiplying the divisor until the dividend was reached, usually by doubling and halving, and of course taking 2/3 wherever possible [2].

1 √1/10 Ans. 1/10	10 1 1	1 √1/2 √1/10 Ans. 1/2 1/10	10 5 1 6
1 1/10 √1/5 Ans. 1/5	10 1 2 2	1 √2/3 √1/30 Ans. 2/3 1/30	10 6 2/3 1/3 7
1 √1/10 √1/5 Ans. 1/10 1/5	10 1 2 3	1 √2/3 √1/10 √1/30 Ans. 2/3 1/10 1/30	10 6 2/3 1 1/3 8
1 2/3 √1/3 √1/15 Ans. 1/3 1/15	10 6 2/3 3 1/3 2/3** 4	1 √2/3 √1/5 √1/30 Ans. 2/3 1/5 1/30	10 6 2/3 2 1/3 9
1 √1/2 Ans. 1/2	10 5 5		

** Written probably as 3 30 1/3, meaning that, since 3×10=30, then 1/30×10=1/3, and hence 1/15×10=2/3.

I follow these by his abbreviated proofs as given in the R. M. P. (except, of course, those for 3, 4, and 5, which I reconstruct) so that one can see the order and regularity with which the Egyptian scribe sets down his work. It is a matter of wonderment to me that an Egyptian scribe could perform such arithmetical operations with the limited tools at his disposal.

1 1/10 √2 1/5 4 1/3 1/15 √8 2/3 1/10 1/30 Total 1	1 1/2 1/10 √2 1 1/5 4 2 1/3 1/15 √8 4 2/3 1/10 1/30 Total 6
1 1/5 √2 1/3 1/15 4 2/3 1/10 1/30 √8 1 1/3 1/5 1/15 Total 2	1 2/3 1/30 √2 1 1/3 1/15 4 2 2/3 1/10 1/30 √8 5 1/2 1/10 Total 7
1 1/5 1/10 √2 1/2 1/10 4 1 1/5 √2 2 1/3 1/15 Total 3	1 2/3 1/10 1/30 √2 1 1/2 1/10 4 3 1/5 √8 6 1/3 1/15 Total 8
1 1/3 1/15 √2 2/3 1/10 1/30 4 1 1/2 1/10 √8 3 1/5 Total 4	1 2/3 1/5 1/30 √2 1 2/3 1/10 1/30 4 3 1/2 1/10 √8 7 1/5 Total 9
1 1/2 √2 1 4 2 √8 4 Total 5	

Summary of the Scribe's Addition of Unit Fractions

From the Recto of the R.M.P.

2÷ 5=1/3 1/15

2÷15=1/10 1/30

2/3 1/5 1/10 1/30=1

Problem 1

1÷10=(2/3 1/5 1/10 1/30)=1

Problem 2

2÷10=1 1/3 1/3 1/5 1/15 1/15
=1 2/3 1/5 (2÷15)
=1 (2/3 1/5 1/10 1/30)=2

3÷10=1 1/3 2/3 1/3 1/5 1/5 1/10 1/15 1/15 1/30
=2 1/3 (2÷5) 1/10 1/15 1/15 1/30
=2 1/3 (1/3 1/15) 1/10 1/15 1/15 1/30
=2 2/3 (1/15 1/15 1/15) 1/10 1/30
=2 (2/3 1/5 1/10 1/30)=3

4÷10=2 2/3 2/3 1/3 (1/10 1/10) 1/15 (1/30 1/30)
=(2 2/3 1/3) 2/3 1/5 1/15 1/15
=3 2/3 1/5 (2÷15)
=3 (2/3 1/5 1/10 1/30)=4

5÷10=1 4=5

Problem 3

6÷10=4 1 (2/3 1/5 1/10 1/30)=6

Problem 4

7÷10=5 1 (1/3 1/3) 1/5 (1/15 1/15)
=6 2/3 1/5 (2÷15)
=6 (2/3 1/5 1/10 1/30)=7

Problem 5

8÷10=(6 1) 1/2 (1/3 1/10 1/15)
=7 1/2 1/2=8

Problem 6

9÷10=7 1 (2/3 1/5 1/10 1/30)=9

A dispassionate view of this array of unit fractions, with its order and symmetry, and indeed its elegance and simplicity (if one considers the mathematical issues involved), leaves one with a feeling of hopeless admiration and wonderment at what was achieved by the Egyptian scribe with the rudimentary arithmetical tools at his disposal.

Notes

1 Following the nomenclature of Chace, Bull, Manning, *The Rhind Mathematical Papyrus* (Oberlin, Ohio: Mathematical Association of America, 1929).

2 See Gillings, "The Egyptian 2/3 table for fractions," *Australian Journal of Science*, XXII (December, 1959), 247.

3 It is convenient to use the plus sign at this stage. The Egyptian scribe had no such notation; mere juxtaposition indicated addition. I also use the signs = and ÷ which also do not have equivalents in the R. M. P., although in the Egyptian Leather Roll

meaning "this is," is repeatedly used.

4 See Gillings, "The division of 2 by the odd numbers 3 to 101 from the Recto of the R. M. P. (B.M. 10058)," *Australian Journal of Science*, XVIII (October, 1955), 43–49.

5 The scribe is familiar with many alternatives evident throughout the R. M. P., and he uses the values 7÷10=1/2+1/5 in Problem 54, while in his table, and in Problem 4, he uses 7÷10=2/3+1/30.

6. Here again the scribe writes 1/2+1/10 as equivalent to 1/3+1/5+1/15, both arising from the multiplication of 2/3+1/10+1/30 by 2. In Problem 4, the scribe does not have any further doubling, so that there is little advantage in using the shorter form there. Thus 2(1 2/3+1/10+1/30)=2+1 1/3+1/5+1/15 =3+(1/3+1/5+1/15)=3+1/2+1/10.

7 O. Neugebauer, *The Exact Sciences in Antiquity* (Princeton, N.J.: Princeton University Press, 1952), p. 76.

8 An error. "Times 10 loaves," should read "of one loaf."