Reprinted from *The Australian Mathematics Teacher*, Vol. 23, No. 3, November, 1967

MATHEMATICAL FRAGMENT FROM THE KAHUN PAPYRUS, iv, 3

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There is a problem in the *Rhind Mathematical Papyrus* (*R.M.P. No.* 64) which deals with the distribution of ten hekats of barley among ten men in such a way that each man's share differs from his neighbour's by $\frac{1}{8}$ hekat. Expressed in modern phraseology, the problem is

The sum of 10 terms of an arithmetical progression is 10, and the common difference is $\frac{1}{8}$. What are the terms of this series?

The scribe directs the solution to be performed as follows.

The average value of the terms is $10 \div 10 = 1$.

The number of differences is 9, one less than the number of terms.

Find half the common difference, it is $\frac{1}{16}$.

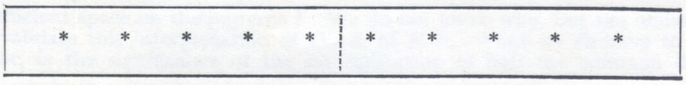
Multiply this by 9, and you get $\frac{9}{16}$.

Add this to the average 1, giving $1\frac{9}{16}$, the highest term.

Now subtract the common difference 9 times, until you reach the lowest term.

Then the series is $1\frac{9}{16}$ $1\frac{7}{16}$ $1\frac{5}{16}$ $1\frac{3}{16}$ $1\frac{1}{16}$ $\frac{15}{16}$ $\frac{13}{16}$ $\frac{11}{16}$ $\frac{9}{16}$ $\frac{7}{16}$.

The reasoning behind the Egyptian scribe's solution is as follows:



There is no middle term. We cannot refer to the $5\frac{1}{2}$th term, but we can speak of half-spaces, or half-common differences. Then from the middle dividing line to either end, there are $4\frac{1}{2}$ spaces, that is, half the total number of spaces, or 9 half common differences, so that if we add 9 times half the common difference to the average value of the terms we obtain the value of the highest term, namely $1\frac{9}{16}$.

Then, by subtracting the common difference, $\frac{1}{8}$, nine times, the required series is found in descending order of magnitude.

In the light of this methodical scribal reasoning, a further interpretation [1] of the mathematical fragment IV, 3, of the *Kahun Papyrus* presents itself. Unlike the scribe of the *R.M.P.*, the scribe of the *K.P.* does not say what problem he is about to solve. He merely shows the following array of numbers and fractions and Egyptologists have long wondered what they signified.

I now suggest that the problem the scribe was solving may have been Problem IV, 3, of the *Kahun Papyrus : The sum of 12 terms of an Arithmetical Progression is 110 and the common difference is $\frac{5}{6}$. What is this series ?*

Then the scribe of the *Kahun Papyrus* may have reasoned along the same lines as the scribe of the *R.M.P.* as follows

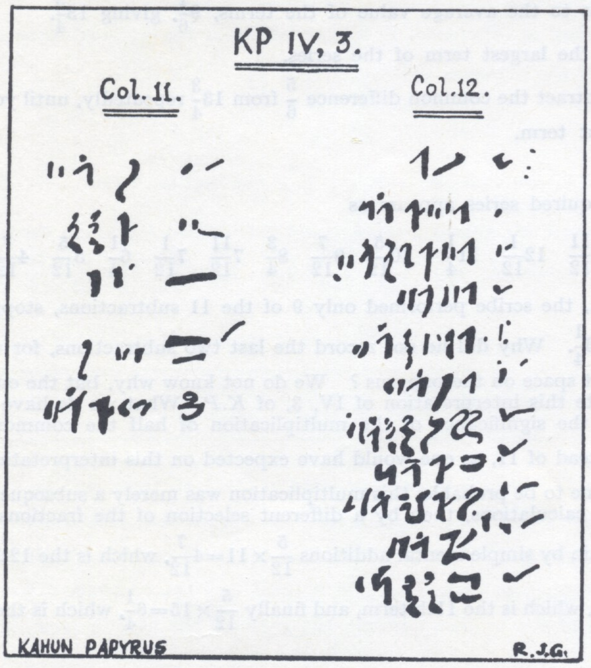
The average value of the 12 terms is $110 \div 12 = 9\frac{1}{6}$. [2]

The number of differences is one less than the number of terms, namely 11.

From the centre to the end there are $5\frac{1}{2}$ spaces, or $5\frac{1}{2}$ differences.

This is the same as 11 half-spaces, or 11 half-differences.

Then multiply half the common difference, namely $\frac{5}{12}$, by 11, giving $4\frac{7}{12}$. [2]



110	\1	$\frac{5}{12}$
$13\frac{3}{4}$	2	$\frac{5}{6}$
$12\frac{11}{12}$	4	$1\frac{2}{3}$
$12\frac{1}{12}$	\8	$3\frac{1}{3}$
$11\frac{1}{4}$	Total	$3\frac{3}{4}$
$10\frac{5}{12}$		
$9\frac{7}{12}$		
$8\frac{3}{4}$		
$7\frac{11}{12}$		
$7\frac{1}{12}$		
$6\frac{1}{4}$		

Add this to the average value of the terms, $9\frac{1}{6}$, giving $13\frac{3}{4}$.

This is the largest term of the series.

Now subtract the common difference $\frac{5}{6}$ from $13\frac{3}{4}$ repeatedly, until you reach the least term.

Then the required series appears as

$$13\frac{3}{4}\quad 12\frac{11}{12}\quad 12\frac{1}{12}\quad 11\frac{1}{4}\quad 10\frac{5}{12}\quad 9\frac{7}{12}\quad 8\frac{3}{4}\quad 7\frac{11}{12}\quad 7\frac{1}{12}\quad 6\frac{1}{4}\quad 5\frac{5}{12}\quad 4\frac{7}{12}.$$

In the *K.P.*, the scribe performed only 9 of the 11 subtractions, stopping at the tenth term, $6\frac{1}{4}$. Why did he not record the last two subtractions, for which there was sufficient space on the papyrus? We do not know why, but the omission does not invalidate this interpretation of IV, 3, of *K.P.* What we *do* have to explain, however, is the significance of the multiplication of half the common difference $\frac{5}{12}$, by 9 instead of 11, as one would have expected on this interpretation. But if, as seems to me to be probable, this multiplication was merely a subsequent check on his previous calculations, then by a different selection of the fractions therein he can also obtain by simple mental additions $\frac{5}{12}\times 11=4\frac{7}{12}$, which is the 12th term, and $\frac{5}{12}\times 13=5\frac{5}{12}$, which is the 11th term, and finally $\frac{5}{12}\times 15=6\frac{1}{4}$, which is the 10th term.

His check marks, and the total he has recorded, give $\frac{5}{12}\times 9=3\frac{3}{4}$, which would be the next term (i.e., the 13th) if the series were extended further, and this is an odd recording for the scribe to have made, which contributed to making *K.P.*, IV, 3, so difficult to interpret.

References

[1] The author first attempted a solution of this problem from the *Kahun Papyrus*, IV, 3, in the early part of 1966, and produced a solution in August, which was published in *The Australian Journal of Science*, Vol. 29, No. 5, in November of that year. Since then, a significant improvement was suggested, by a comparison with Problem 64 of the *Rhind Mathematical Papyrus* which we give here.

[2] Omitting the unit numerators of the Egyptian fractions, and writing $\bar{\bar{3}}$ for $\frac{2}{3}$, abbreviations which are now regarded as standard, we would have

110÷12		
	\ 1	12
	2	24
	4	48
	\ 8	96
	$\bar{3}$	4
	\ $\bar{6}$	2
	$9\bar{6}$	110

$\bar{4}\bar{6}$ × 11		
	\ 1	$\bar{4}\bar{6}$
	\ 2	$\bar{2}\bar{3}$
	4	$1\bar{\bar{3}}$
	\ 8	$3\bar{3}$
	11	$\bar{3}\bar{2}\bar{3}\bar{3}\bar{4}\bar{6}$
		$3\bar{2}\bar{2}\bar{3}\bar{4}$
	11	$4\bar{3}\bar{4}$

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SEAMER & ARUNDEL STS., GLEBE, SYDNEY, 2037