Reprinted from *The Australian Journal of Science*, Vol. 22, No. 6, December, 1959, p. 247.

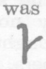
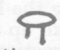
The Egyptian $^2/_3$ Table for Fractions

The Rhind Mathematical Papyrus (B.M. 10057-8)

R. J. Gillings*

1. (1). The mathematics of the Rhind Papyrus (B.M. 10057–8)[1] accords us a view of the earliest known mathematical achievements of the ancient Egyptians. The papyrus was written by a scribe, Ah-mose (he tells us), during the Hyksos period about 1650 B.C., being a copy of writings made some 200 years earlier. It is written in a cursive form of hieroglyphics called hieratic, is written from right to left, page by page so to speak, and contains 87 mathematical problems[2] preceded by a long section (referred to as the Recto), dealing with the division of the number 2 by all the odd numbers[3] from 3 to 101.

1. (2). The Egyptians conceived of numbers as consisting of two kinds, first, all the integers from 1 up to a million, and second, ordinary common fractions with unity as the universal numerator, so that in modern parlance, we can say that they regarded all numbers as being either integers or the reciprocals of integers. They were able, with great facility it appears to us, to add, subtract, multiply and divide these integers and fractions, and the problems proposed and solved in the R.M.P. involve all these and other operations.

1. (3). The one remarkable exception to the fractions with unit numerators was $^2/_3$,[4] which was written with a special sign  in hieratic and  in hieroglyphics. The scribe used the fraction $^2/_3$ as an operator so freely in his multiplications and divisions that one is led to believe with Peet[5] that he must have used a prepared table, a lot of which he probably knew by heart, just as we know our multiplication tables to-day. Further, this $^2/_3$ table was so much a part of an Egyptian scribe's stock-in-trade that, should he require to find $^1/_3$ of an integer or fraction, he would first find $^2/_3$ of it and then halve his answer, instead of merely dividing the number by 3. It is the purpose of this article to show how, with the mathematical tools at his disposal, the Egyptian scribe could have produced such a table, since, except for an interesting rule given in Problem 61B, and a short table for $^2/_3$ of eight simple fractions in Problem 61, no explanation of his methods of obtaining $^2/_3$ of *any* number, is given in the R.M.P.[6]

* Sydney Teachers' College.

[1] Hereafter referred to as the R.M.P. (B.M. 10057-8), is the British Museum catalogue number.

[2] Adopting the numbering given by Chace (1927), *The Rhind Mathematical Papyrus* (Math. Assoc. of America, Oberlin, Ohio).

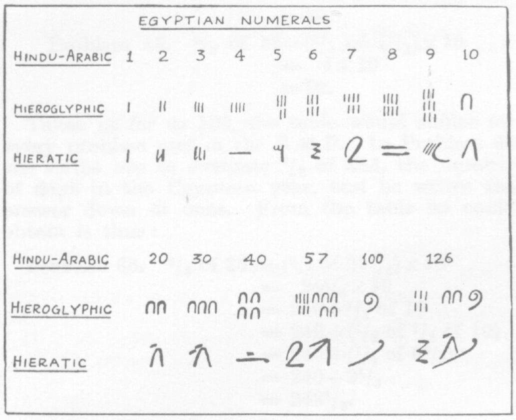
See my *Division of 2 by the Odd Numbers* (*Aust. J. Sci.*, *18*, 43 (1955)).

[4] Here, as elsewhere, I write the Egyptian fractions as we write them to-day for ease in printing. The reader must keep in mind that in the R.M.P. all unit fractions are merely integers with dots above them.

[5] T. E. Peet, *The Rhind Mathematical Papyrus* (London, 1923).

[6] Nor elsewhere. Peet (1923, 2), p. 6, says that the Moscow (or Golenishchev) papyrus 'contains nothing with the exception of the truncated pyramid, which will greatly modify the conception of Egyptian mathematics given to us by the already published papyri'. Chace, *R.M.P.* (1927).

1. (4). For writing integers the Egyptians had a well-defined decimal notation, but without the place-value of the Hindu-Arabic system which we use to-day, and no zero. Thus they wrote,



Juxtaposition of signs meant simple addition, just as 3 plus a quarter is written $3\frac{1}{4}$ by us to-day. The Egyptians had no signs for plus or minus nor for multiplication or division. For writing fractions, they used exactly the same signs as they used for integers, but with the addition of the Horus-Eye ⬭ over the hieroglyphic numbers, and a single 'dot' made with the brush or pen, over the hieratic numbers. This explains why with their notation such fractions as $^2/_7$ or $^5/_{12}$ could not be written by them, but were put down as $(^1/_4+^1/_{28})$ and $(^1/_3+^1/_{12})$, of course without the plus signs. Nor would the scribe ever write $(^1/_7+^1/_7)$ for $^2/_7$ and certainly not $(^1/_{12}+^1/_{12}+^1/_{12}+^1/_{12}+^1/_{12})$ for $^5/_{12}$. He thought, apparently, as we do in our ordinal numbers, of the seventh part being the last part, which goes with the other six to make up the whole, and so also with all other unit fractions.[7]

1. (5). Addition and subtraction of integers presented no difficulty. For fractions, his methods were analogous to our present methods of using L.C.M., but with this difference: the Egyptian would use any convenient common multiple that suggested itself, and preferably an abundant number.[8] Thus for example in Problem 37, (R.M.P.), he writes: $(^1/_{72}+^1/_{16}+^1/_{32}+^1/_{64}+^1/_{576})$, taken as parts of 576 are $(8+36+18+9+1)$, a total of 72, and thus $^1/_8$ of 576. His answer is therefore $^1/_8$. Again in Problem 22, he writes: $(^2/_3+^1/_5+^1/_{10}+^1/_{30})=1$, for applied to 30, these fractions are equal to $20+6+3+1$, making 30.

1. (6). Multiplication was performed by constant doubling and sometimes multiplying by 10, as for example, Problem 70, 2801×7

If	*1	is	2801
then	*2	is	5602
and	*4	is	11204
Total	7	is	19607

the asterisks[9] denoting that these are the numbers to be added. Multiplication with fractions was set out in the same way, as for example in Problem 19, where the scribe wishes to multiply together $^1/_{12}$ and $(1+^2/_3+^1/_3)$.

If	*1	is	$^1/_{12}$
then	*$^2/_3$	is	$^1/_{18}$
and	*$^1/_3$	is	$^1/_{36}$
Total	$(1+^2/_3+^1/_3)$	is	$^1/_6$

since applied to 18, $(^1/_{12}+^1/_{18}+^1/_{36})$ become $(1^1/_2+1+^1/_2)$, or 3, which is $^1/_6$ of 18, and thus $^1/_6$ is the answer he seeks.

1. (7). Subtraction is along similar lines, as for example Problem 21, 'complete $(^2/_3+^1/_{15})$ to 1', by which the scribe means evaluate $1-(^2/_3+^1/_{15})$, a problem in '*Completion*'. He proceeds thus:

Applied to 15, $^2/_3$ is 10 and $^1/_{15}$ is 1, making a total of 11. Now (15—11) is 4, needed for completion. Then multiply 15 so as to get 4.

If	1	is	15
then	$^1/_{10}$	is	$1^1/_2$
and	*$^1/_5$	is	3
and	*$^1/_{15}$	is	1
total	$(^1/_5+^1/_{15})$	is	4.

Therefore $(^1/_5+^1/_{15})$ is what must be added to complete $(^2/_3+^1/_{15})$ to 1.

1. (8). Division. To evaluate, 19 divided by 114, the scribe proceeds:

If	1	is	19	
then	*2	is	38	
and	*4	is	76	
therefore	6	is	114	$^1/_6$

by which he means that, because $6\times 19=114$, then $^1/_{114}$ of $19=^1/_6$. But division was usually performed by successive multiplication of the divisor until the dividend was reached. If ordinary multiplication did not exactly produce the required dividend, the scribe found the difference and performed again on that number separately. They even had short division tables, and towards the end of the recto the scribe includes a list of the division of the numbers 1 to 9, divided by 10, as, for example, $4\div 10=(^1/_3+^1/_{15})$, $7\div 10=(^2/_3+^1/_{30})$ and $9\div 10=(^2/_3+^1/_5+^1/_{30})$.

2. (1). A careful check of all the problems in the R.M.P. shows that there are at least 24 examples of the scribe writing $^2/_3$ of integers in one simple operation, which the following will illustrate.

The Recto.	$^2/_3$ of	27	is	18
Problem 66.	$^2/_3$ of	365	is	$243^1/_3$
Problem 33.	$^2/_3$ of	5432	is	$3621^1/_3$.

There are, further, at least 14 examples in which the scribe writes down immediately $^2/_3$ of fractional numbers, such as

Problem 67.	$^2/_3$ of	$^1/_3$	is	$(^1/_6+^1/_{18})$.
Problem 70.	$^2/_3$ of	$(7^1/_2\ ^1/_4\ ^1/_8)$	is	$5^1/_4$.
Problem 30.	$^2/_3$ of	$13^1/_{23}$	is	$8^2/_3\ ^1/_{46}\ ^1/_{138}$).

[7] Compare B. L. Van der Waerden, *Science Awakening* (English edition, 1954. Noordhoff, Groningen, Holland), p. 20. 'The fourth part presents quite naturally a concrete image: three parts and then a fourth part combine to make a whole.'

[8] An abundant number is one, the sum of whose divisors exceeds the number itself, as 12, 18, 20, 24, 30, 36, . . ., etc.

[9] We use an asterisk here. The scribe used a sloped check mark.

There are also at least 18 examples in which to obtain $^1/_3$ of a number ; the scribe first takes $^2/_3$ of it and then halves his answer, as for example

Problem 43. To get $^1/_3$ of 8, $^2/_3$ of it is $5^1/_3$,
hence $^1/_3$ of it is $2^2/_3$.

Problem 67. To get $^1/_3$ of 315, $^2/_3$ of it is 210,
hence $^1/_3$ of it is 105.

Problem 38. To get $^1/_3$ of 320, $^2/_3$ of it is $213^1/_3$,
hence $^1/_3$ of it is $106^2/_3$.

There are 17 further examples where the scribe writes $^1/_3$ of a fraction or a mixed number, sometimes omitting the customary intermediate $^2/_3$ step.

Problem 36. To get $^1/_3$ of ($^1/_4$ $^1/_{53}$ $^1/_{106}$ $^1/_{212}$),
it is ($^1/_{12}$ $^1/_{159}$ $^1/_{318}$ $^1/_{636}$).

Problem 37. To get $^1/_3$ of ($^1/_4$ $^1/_{32}$) it is ($^1/_{12}$ $^1/_{96}$).

2. (2). That the Egyptian had a $^2/_3$ table at his disposal is evidenced by the short table listed as Problem 61B in the R.M.P. There are given a few simple examples of $^2/_3$ of certain fractions, possibly to be learnt off by heart by pupils. It is headed *Table for Multiplication of Fractions*, and contains 17 entries, from which we choose the following as examples.

$^2/_3$ of $^2/_3$ is ($^1/_3+^1/_9$)
$^2/_3$ of $^1/_3$ is ($^1/_6+^1/_{18}$)
$^2/_3$ of $^1/_2$ is $^1/_3$
$^2/_3$ of $^1/_7$ is ($^1/_{14}+^1/_{42}$)
$^2/_3$ of $^1/_9$ is ($^1/_{18}+^1/_{54}$).

2. (3). It is appropriate at this stage to include some brief examples of tables for the addition of fractions, taken from the Egyptian Leather Scroll (B.M. 10250),[10] which was found with the R.M.P. at the Ramesseum at Thebes. It is dated about 1700 B.C.

Columns 3 and 4.[10]
$^1/_{30}+^1/_{45}+^1/_{90}=^1/_{15}$
$^1/_{24}+^1/_{48}=^1/_{16}$
$^1/_{18}+^1/_{36}=^1/_{12}$
$^1/_{21}+^1/_{42}=^1/_{14}$
etc., etc.

Columns 5 and 6.
$^1/_{10}+^1/_{40}=^1/_8$
$^1/_4+^1/_{12}=^1/_3$
$^1/_{25}+^1/_{15}+^1/_{75}+^1/_{200}=^1/_8$
$^1/_{50}+^1/_{30}+^1/_{150}+^1/_{400}=^1/_{16}$
etc., etc.

Columns 1 and 2 are partly destroyed, but one can safely read here $^1/_{10}+^1/_{40}=^1/_8$, (a repetition), and [$^1/_4$]$+^1/_{12}=^1/_3$ and $^1/_5+^1/_{20}=^1/_4$, also repetitions.

2. (4). There is abundant evidence that the Egyptians were fully aware of the operation which in modern notation we would write as

If $a\times b=c$
then $1/c\times b=1/a$.

Thus in the recto alone there are approximately 80 examples of the following type

because $12\times 23=276$
then $^1/_{276}$ of $23=^1/_{12}$

because $7\times 97=679$
then $^1/_{679}$ of $97=^1/_7$ etc., etc.

3. (1). We now show how, using the arithmetical operations of which he was capable, the scribe could have prepared a comprehensive $^2/_3$ table, and almost certainly did have some portions of it, for ready reference.

First, he can easily form a $1^1/_2$ times table of integers as under

$1^1/_2$ of 1 is $1^1/_2$
$1^1/_2$ of 2 is 3
$1^1/_2$ of 3 is $4^1/_2$
$1^1/_2$ of 4 is 6
$1^1/_2$ of 5 is $7^1/_2$
$1^1/_2$ of 6 is 9
$1^1/_2$ of 7 is $10^1/_2$
$1^1/_2$ of 8 is 12 etc., etc.

These numbers form two simple series in arithmetical progression[11] which he can continue as far as he pleases. Now, by re-writing this table in the way with which he is familiar, as noted in **2.** (4), he can write, as far as he pleases, the table

1 is $^2/_3$ of $1^1/_2$
2 is $^2/_3$ of 3
3 is $^2/_3$ of $4^1/_2$
4 is $^2/_3$ of 6
5 is $^2/_3$ of $7^1/_2$
6 is $^2/_3$ of 9
7 is $^2/_3$ of $10^1/_2$
8 is $^2/_3$ of 12 etc., etc.

The scribe can now insert between each consecutive pair of the series in the right-hand column, two equally spaced numbers, as 2 and $2^1/_2$ between the first two, producing an even simpler A.P., with a common difference of $^1/_2$. A similar operation on the left-hand column produces the corresponding A.P., with a common difference of $^1/_3$. Thus his projected $^2/_3$ table takes the following form, which he can write just as far as he pleases.

$^2/_3$ of $^1/_2$	is $^1/_3$	Prob. 61.
$^2/_3$ of 1	is $^2/_3$	Prob. 67.
$^2/_3$ of $1^1/_2$	is 1	
$^2/_3$ of 2	is $1^1/_3$	
$^2/_3$ of $2^1/_2$	is $1^2/_3$	
$^2/_3$ of 3	is 2	Prob. 25.
$^2/_3$ of $3^1/_2$	is $2^1/_3$	Prob. 69.
$^2/_3$ of 4	is $2^2/_3$	
$^2/_3$ of $4^1/_2$	is 3	
$^2/_3$ of 5	is $3^1/_3$	Prob. 46.
$^2/_3$ of $5^1/_2$	is $3^2/_3$	
$^2/_3$ of 6	is 4	Prob. 39.
$^2/_3$ of $6^1/_2$	is $4^1/_3$	
$^2/_3$ of 7	is $4^2/_3$	
$^2/_3$ of $7^1/_2$	is 5	Prob. 70.
$^2/_3$ of 8	is $5^1/_3$	Prob. 43.
		etc., etc.

3. (2). We can now show that for many of the problems which come to the scribe's attention, the above table, taken as far as $^2/_3$ of 10, will in general suffice for his needs, if we recall [**1.** (6)] that

[10] I designate the columns reading from right to left, but write them here from left to right, adding plus and equal signs for ease in reading. Reference here to the Egyptian Leather Scroll is to show that use of tables was not uncommon. Indeed the scroll contains nothing else but tables like those shown here.

[11] The Egyptians were quite familiar with progressions. In Problem 64, Ah-mose deals with an A.P. whose common difference is $^1/_8$, and in Problem 79 he finds the sum of five terms of a G.P. whose common ratio is 7. Indeed, this latter is by some thought to be the original of the nursery rhyme 'As I was going to St. Ives . . .'. Compare Chase R.M.P.

multiplication by 10 was a commonplace operation in integers, requiring merely a change in symbols.

$$\text{Problem 40.}\quad {}^{2}/_{3} \text{ of } 60 = ({}^{2}/_{3} \text{ of } 6)\times 10 = 4\times 10 = 40.$$

$$\text{Problem 45.}\quad {}^{2}/_{3} \text{ of } 15 = ({}^{2}/_{3} \text{ of } 1{}^{1}/_{2})\times 10 = 1\times 10 = 10.$$

Taken as far as 100, the table would suffice for every problem met in the R.M.P. In Problem 66 the scribe has to evaluate ${}^{2}/_{3}$ of 365, the number of days in the Egyptian year, and he writes the answer down at once. From the table he could obtain it thus:

$$\begin{aligned}\text{Problem 66.}\quad {}^{2}/_{3} \text{ of } 365 &= ({}^{2}/_{3} \text{ of } 36{}^{1}/_{2})\times 10\\ &= 24{}^{1}/_{3}\times 10\\ &= 240+{}^{1}/_{3} \text{ of } 10\\ &= 240+({}^{1}/_{2} \text{ of } {}^{2}/_{3} \text{ of } 10)\\ &= 240+({}^{1}/_{2} \text{ of } 6{}^{2}/_{3})\\ &= 240+3{}^{1}/_{3}\\ &= 243{}^{1}/_{3}.\end{aligned}$$

3. (3). For fractions other than those in the table, the scribe has a rule which he gives explicitly in Problem 61B without proof:

'*To get ${}^{2}/_{3}$ of the reciprocal of any odd number, take the reciprocals of twice the number and the reciprocal of six times the number.*'

In modern notation we would write

$$\begin{aligned}{}^{2}/_{3} \text{ of } 1/n &= 2/3n\\ &= 4/6n\\ &= (3+1)/6n\\ &= 1/2n+1/6n.\end{aligned}$$

Of course, this formula holds for n any integer, but the scribe only uses it for odd values of n.[12] For even values of n the scribe has a much simpler method. Since he knows that ${}^{2}/_{3}$ is the reciprocal of $1{}^{1}/_{2}$, he can use the process which in modern notation would be written

$$\begin{aligned}&{}^{2}/_{3} \text{ of } 1/n = 1/(1{}^{1}/_{2} \text{ of } n)\\ \text{and thus }&{}^{2}/_{3} \text{ of } {}^{1}/_{12} = 1/1{}^{1}/_{2} \text{ of } 12\\ &= {}^{1}/_{18}. \qquad \text{Problem 19.}\end{aligned}$$

This is merely adding half the even number to itself and regarding it as a fraction.[13]

3. (4). In Problem 42, ${}^{2}/_{3}$ of $(8+{}^{2}/_{3}+{}^{1}/_{6}+{}^{1}/_{18})$ is written down at once as $(5+{}^{2}/_{3}+{}^{1}/_{6}+{}^{1}/_{18}+{}^{1}/_{27})$. From his ${}^{2}/_{3}$ tables this answer can be derived as follows:

$$\begin{aligned}&{}^{2}/_{3} \text{ of } (8+{}^{2}/_{3}+{}^{1}/_{6}+{}^{1}/_{18})\\ &= ({}^{2}/_{3} \text{ of } 8)+({}^{2}/_{3} \text{ of } {}^{2}/_{3})+({}^{2}/_{3} \text{ of } {}^{1}/_{6})+({}^{2}/_{3} \text{ of } {}^{1}/_{18})\\ &= 5{}^{1}/_{3} + ({}^{1}/_{3}+{}^{1}/_{9}) + {}^{1}/_{9} + {}^{1}/_{27}\\ &\quad \text{[table.]} \quad \text{[2. (2).]} \quad \text{[3. (3).]} \quad \text{[3. (3).]}\\ &= 5{}^{2}/_{3} + 2\div 9 + {}^{1}/_{27}\\ &= 5{}^{2}/_{3} + ({}^{1}/_{6}+{}^{1}/_{18}) + {}^{1}/_{27}\\ &\qquad\qquad \text{[Recto]}\end{aligned}$$

Again, in Problem 32 it is required to find ${}^{1}/_{3}$ of $(1+{}^{1}/_{3}+{}^{1}/_{4})$. The scribe first finds ${}^{2}/_{3}$ of this number and then halves his answer.

$$\begin{aligned}&\text{To get } {}^{1}/_{3} \text{ of } (1+{}^{1}/_{3}+{}^{1}/_{4})\\ &{}^{2}/_{3} \text{ of it is } = {}^{2}/_{3}+({}^{1}/_{6}+{}^{1}/_{18})+{}^{1}/_{6}\\ &\qquad \text{[table] [Prob. 61B] [3. (3).]}\\ &\qquad = {}^{2}/_{3}+{}^{1}/_{3}+{}^{1}/_{18}\\ &\qquad = 1+{}^{1}/_{18}\\ &\text{therefore } {}^{1}/_{3} \text{ is } = {}^{1}/_{2}+{}^{1}/_{36}.\end{aligned}$$

3. (5). It is possible to explain in this manner every one of the numerous instances of the scribe's capabilities with ${}^{2}/_{3}$ or ${}^{1}/_{3}$ of any numbers. The use of a ${}^{2}/_{3}$ table was so much a part of the scribe's methods of working that we find such remarkable examples as the following:

Problem 25. To get ${}^{1}/_{3}$ of 3.

If 1 is 3
then ${}^{2}/_{3}$ is 2
hence ${}^{1}/_{3}$ is 1.

If to our modern minds this seems incredibly protracted for such a simple problem, what are we to think of this example?

Problem 67. To get ${}^{1}/_{3}$ of 1.

If 1 is 1
then ${}^{2}/_{3}$ is ${}^{2}/_{3}$
hence ${}^{1}/_{3}$ is ${}^{1}/_{3}$.

Clearly, the scribe has an overwhelming trust in the efficacy of his ${}^{2}/_{3}$ table. And well he might when it is required, as in Problem 33, to get ${}^{2}/_{3}$ of $(16+{}^{1}/_{56}+{}^{1}/_{679}+{}^{1}/_{776})$, a piece of mechanical arithmetic to daunt any modern schoolboy. He writes the answer at once as

$$(10+{}^{2}/_{3}+{}^{1}/_{84}+{}^{1}/_{1358}+{}^{1}/_{4074}+{}^{1}/_{1164}).$$

Let us see how easy this really is to the scribe with his ${}^{2}/_{3}$ tables.

The answer is

$$\begin{aligned}&({}^{2}/_{3} \text{ of } 16)+({}^{2}/_{3} \text{ of } {}^{1}/_{56})+({}^{2}/_{3} \text{ of } {}^{1}/_{679})+({}^{2}/_{3} \text{ of } {}^{1}/_{776})\\ &= 10{}^{2}/_{3} + {}^{1}/_{84} + {}^{1}/_{1358}+{}^{1}/_{4074}+{}^{1}/_{1164}.\\ &\quad \text{[table]} \quad \text{[3. (3).]} \quad \text{[Prob. 61B]} \quad \text{[3. (3).]}\end{aligned}$$

3. (6). To the student of ancient Egyptian mathematics as revealed in the Rhind Mathematical Papyrus, neither the method of approximating to the area of a circle ($\pi = {}^{256}/_{81}$), nor the method of multiplication of two numbers by mere doubling, is as astounding as the comparative ease and facility with which ${}^{2}/_{3}$ or ${}^{1}/_{3}$ of an apparently complex fractional number is achieved. It is this situation which inspired this article. One has only to look at the following, chosen more or less at random, to see that an urge to explain them away would well exercise any inquiring mind.

$$\begin{aligned}{}^{2}/_{3} \text{ of } 7+{}^{1}/_{2}+{}^{1}/_{4}+{}^{1}/_{8} &= 5{}^{1}/_{4}.\\ {}^{2}/_{3} \text{ of } 10{}^{2}/_{3} &= 7{}^{1}/_{9}.\\ {}^{2}/_{3} \text{ of } 13{}^{1}/_{23} &= 8+{}^{2}/_{3}+{}^{1}/_{46}+{}^{1}/_{138}.\\ {}^{2}/_{3} \text{ of } 16+{}^{1}/_{56}+{}^{1}/_{679}+{}^{1}/_{776} &= 10+{}^{2}/_{3}+{}^{1}/_{1358}+\\ &\quad {}^{1}/_{4074}+{}^{1}/_{1164}\\ {}^{1}/_{3} \text{ of } {}^{1}/_{4}+{}^{1}/_{32} &= {}^{1}/_{12}+{}^{1}/_{96}.\\ {}^{1}/_{3} \text{ of } 1+{}^{1}/_{3}+{}^{1}/_{4} &= {}^{1}/_{2}+{}^{1}/_{36}.\\ {}^{1}/_{3} \text{ of } 8+{}^{2}/_{3}+{}^{1}/_{6}+{}^{1}/_{18} &= 2+{}^{2}/_{3}+{}^{1}/_{6}+{}^{1}/_{12}+\\ &\quad {}^{1}/_{32}+{}^{1}/_{54}.\\ {}^{1}/_{3} \text{ of } 1+{}^{1}/_{6}+{}^{1}/_{12}+{}^{1}/_{114}+{}^{1}/_{228} &= {}^{1}/_{3}+{}^{1}/_{18}+{}^{1}/_{36}+\\ &\quad {}^{1}/_{342}+{}^{1}/_{684}.\end{aligned}$$

---

[12] In Problem 61, the table referred to in 2. (2), there is one entry which reads ${}^{2}/_{3}$ of ${}^{1}/_{6} = {}^{1}/_{12}+{}^{1}/_{36}$, an example of his rule in Problem 61B applied to an even number. Such usage is rare, however it is certainly the only one known to me.

[13] As Chace remarks (*R.M.P.*, p. 25), all that is necessary is 'to take $1{}^{1}/_{2}$ times the number and dot it'.

AUSTRALASIAN MEDICAL PUBLISHING CO. LTD.
SEAMER AND ARUNDEL STS., GLEBE SYDNEY