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The Division of 2 by the Odd Numbers 3 to 101 from the Recto of the Rhind Mathematical Papyrus (B.M. 10058)

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1.1 The recto of the Rhind Mathematical Papyrus in the British Museum is the first half of the roll, whose overall length is 18 feet and whose height is a little more than one foot. It was written by a scribe, Ah-mose, during the period of the Hyksos or Shepherd Kings of Egypt, about 1650 B.C., and is a copy of writings made about 200 years earlier. The papyrus is written in hieratic, a cursive form of hieroglyphics, and reads from right to left. It contains some 87 problems[1] of a mathematical nature, preceded by a rather long section dealing with the division of 2 by all the odd numbers from 3 to 101. This section occupies almost one-third of the whole papyrus. It is this portion of the R.M.P. which we are to discuss. For his introduction the scribe writes:[2]

> Accurate reckoning. The entrance into the knowledge of all existing things, and all obscure secrets. This bock was copied in the year 33, in the 4th month of the inundation season, under the majesty of the king of Upper and Lower Egypt, A-user-Re, endowed with life, in likeness to writings of old, made in the time of the king of Upper and Lower Egypt, Ne-ma-et-Re. It is the scribe Ah-mose who copies this writing.

1.2 Since the Egyptian mathematician performed all his multiplications by repeated doubling and halving, he had to be able to double any number integral or fractional, and the doubling of fractions was by far the more difficult. This was so, because with his notation, in which a dot placed on top of a number turned it into its reciprocal, every Egyptian fraction had unity for its numerator, with the sole and remarkable exception of 2/3, for which he had a special sign. The doubling of even unit fractions like 1/24, he could immediately write as 1/12, but the doubling of odd unit fractions like 1/17 was quite a different matter, because the answer could only be expressed in terms of other unit fractions. Suppose for example he desires to double 1/9. There are many solutions from which he may choose, such as[3] 1/5 + 1/45, 1/6 + 1/18, 1/8 + 1/12 + 1/72, 1/9 + 1/10 + 1/90, or 1/9 + 1/12 + 1/36, and so on indefinitely. But the scribe usually chooses the simplest answer available, using the least number of fractions possible, consistent with small numbers. In no case, in the division of 2 by the odd numbers, does the scribe accept an answer with a denominator as large as 1000. Nor is the answer 1/9 + 1/9 acceptable to the Egyptian mathematician, and he seldom wrote two equal unit fractions together in that way.[4]

1.3 In this part of the recto, in which the scribe treats the division of 2 by odd numbers, he first sets down his answer and then proves by the arithmetical means at his command that it is correct. In other words, the answers are given first and the proofs follow, after the manner of Euclid's 'Elements'. Authorities, however, are not all agreed on this point. Chace for example[5] regards the scribe's calculations partly as solutions, as well as proofs, while Hultsch[6] and Peet[7] consider them to be proofs only. Hultsch says there is indeed no indication of his methods at all. The general problem of expressing the proper fraction p/q as the sum of unit fractions has been studied by many mathematicians, notably Sylvester (1880), Collignon (1881), Tannery (1884), Mansion (1888), Loria (1894), Bobynin (1899), Hultsch (1901), Simon (1907), Vetter (1922), Vasconcellos (1923)[8] Eisenlohr had shown in 1877 that some of the results of the table could be obtained from the formula

$$\frac{2}{n} = \frac{1}{1/2(n+1)} + \frac{1}{1/2n(n+1)},$$

but only four of the fifty answers given are obtainable from this formula, namely, 2/5, 2/7, 2/11 and 2/23. Chace says, 'no formula or rule has been discovered that will give all the results of the table, and Loria expressly states that he does not attempt to indicate how the old Egyptians obtained them'.[9] Neugebauer discussed these fractions in some detail,[10] and Archibald says of his work, 'This monograph shows considerable insight into the spirit of Egyptian mathematics; but it does not appear to indicate the most natural explanation of th way in which the table was formed.'[11]

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1.4 I propose to explain how the Egyptian scribe could have obtained all the answers given in his table, by a regular and consistent procedure, in conformity with the mathematical operations of which the scribe was capable. In this procedure, the values given for 2 divided by 5, 7, 11 and 23, which are obtainable from the formula suggested by Eisenlohr, will be found to be included merely as special cases of a more general method.

For convenience, we consider the fifty odd numbers from 3 to 101 as divided into three groups, group I containing all the multiples of 3, group II containing 5 and 7 and their multiples not already considered in group I, and group III containing all the remaining primes. Thus we shall have,

Group I: 3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, 99.

Group II: 5, 25, 35, 55, 65, 85, 95, 7, 49, 77, 91.

Group III: 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101.

2.1 *Group I, 2 divided by multiples of 3.*

In problem 61B[12] the scribe states his rule for finding 2/3 of the reciprocal of any odd number:

> Make thou times of it 2, times of it 6. 2/3 of it this is. Behold does one according to the like for fractions every uneven, which may occur.

Then he gives the example:[3]

$$2/3 \text{ of } 1/5 = \frac{1}{2 \times 5} + \frac{1}{6 \times 5} = 1/10 + 1/30$$

The rule is of course true for even numbers also, and he sometimes so uses it, as in problem 61, where he writes, 2/3 of 1/6 = 1/12 + 1/36; but this is not usual. In problem 33 the scribe writes: 2/3 of 1/1679 = 1/1358 + 1/4074; an excellent illustration of the use of this rule. In modern notation, for n any integer,

$$\begin{aligned} 2/3 \text{ of } 1/n &= 2/3n \\ &= 4/6n \\ &= (3+1)/6n \\ &= 1/2n + 1/6n \end{aligned}$$

2.2 The frequent use of the exceptional 2/3 fraction was so much an important part of Egyptian arithmetic that to find 1/3 of a quantity the scribe would first find 2/3 of it and then halve his answer, a process which to our modern minds appears extraordinary, and more so when we observe, in problem 25, how the scribe solves the simple problem of finding 1/3 of 3. First he notes that 2/3 of 3 is 2, and therefore 1/3 of 3 is the half of 2, namely 1. T. Eric Peet has written:[7]

> The Egyptian scribe was capable of taking 2/3 of a number in a single process, which is equivalent to saying that he used the 2/3 times table, and probably knew it by heart. Stranger still, he obtained 1/3 of a quantity, not by dividing it by 3, but by halving 2/3 of it.

2.3 For his first number, 2 divided by 3, the scribe could have used this formula and written 2/3 of 1/1 = 1/2 + 1/6, but he merely puts 2/3. In later divisions,[13] where the working calls for 1/2 + 1/6, he writes 2/3 at once, as being too trivial to require explanation. Then for all the multiples of 3 he can at once write the equivalent of[3]

2/9	= 2/3 of 1/3	= 1/6	+	1/18
2/15	= 2/3 of 1/5	= 1/10	+	1/30
2/21	= 2/3 of 1/7	= 1/14	+	1/42
2/27	= 2/3 of 1/9	= 1/18	+	1/54
2/33	= 2/3 of 1/11	= 1/22	+	1/66
2/39	= 2/3 of 1/13	= 1/26	+	1/78
2/45	= 2/3 of 1/15	= 1/30	+	1/90
2/51	= 2/3 of 1/17	= 1/34	+	1/102
2/57	= 2/3 of 1/19	= 1/38	+	1/114
2/63	= 2/3 of 1/21	= 1/42	+	1/126
2/69	= 2/3 of 1/23	= 1/46	+	1/138
2/75	= 2/3 of 1/25	= 1/50	+	1/150
2/81	= 2/3 of 1/27	= 1/54	+	1/162
2/87	= 2/3 of 1/29	= 1/58	+	1/174
2/93	= 2/3 of 1/31	= 1/62	+	1/186
2/99	= 2/3 of 1/33	= 1/66	+	1/198

In the R.M.P. the text shows exactly these fractions in this pattern, and they do not vary in the slightest.[14] The scribe, aware of the above results, writes down the respective answers first, in his setting out, and then proves that they are correct by finding 2/3 of the number being considered, and then finding what fraction of the number will give 1/2. Since his setting out is the same for the 17 divisions above, we need only look at one of them, and 2 divided by 51 will suffice for illustration.

2 divided by 51

1/34 of 51 is 1 1/2, 1/102 of 51 is 1/2.

1	51	
*2/3	34	1 1/2
*2	102	1/2

By this the scribe means, in line 1, that his answer is 1/34 + 1/102, for 1/34 and 1/102 of 51 are 1 1/2 and 1/2, which is 2. For his proof, which he gives in lines 2, 3 and 4, he says that if one part of the number is 51, then 2/3 of it is 34, and therefore (in the same line) 1/34 of 51 is 1 1/2, for he knows that the reciprocal of 2/3 is 1 1/2.[15] In the last line he writes, 2 times 51 is 102, and therefore 1/102 of 51 is 1/2. He then puts check marks on the lines containing 1 1/2 and 1/2 (here shown by asterisks), indicating that 1 1/2 and 1/2 are to be added. This completes his proof that 2/51 = 1/34 + 1/102.

2.4 To obtain his answers for numbers not multiples of 3, the scribe has a more general method, which although applicable to multiples of 3 also, is perhaps not so rapid for him as the foregoing. For this method the scribe gives an adumbration in 2 divided by 35, and this is the *only* division of the whole fifty in which any indication of his methods is given at all. In his setting down for this division the scribe adds three extra numbers which he used as multipliers.

2 divided by 35

35.	1/30 of 35 is 1 1/6,	1/42 of 35 is 2/3 1/6.	
6	7	5	
	1		35
	*1/30		1 1/6
	*1/42		2/3 1/6

that their sum is unity. Again in problem 37 he writes—

in black,	1/72,	1/16,	1/32,	1/64,	1/576—	1/8
in red,	8,	36,	18,	9,	1—	72

showing that he has applied the numbers to 576, and written the answers in contrasting colours below, and added them to obtain 72, which is 1/8 of 576.

In all the examples in the R.M.P., where the scribe adopts this procedure, the numbers he chooses have one property in common. They are all *abundant numbers*[16] like the 30 and

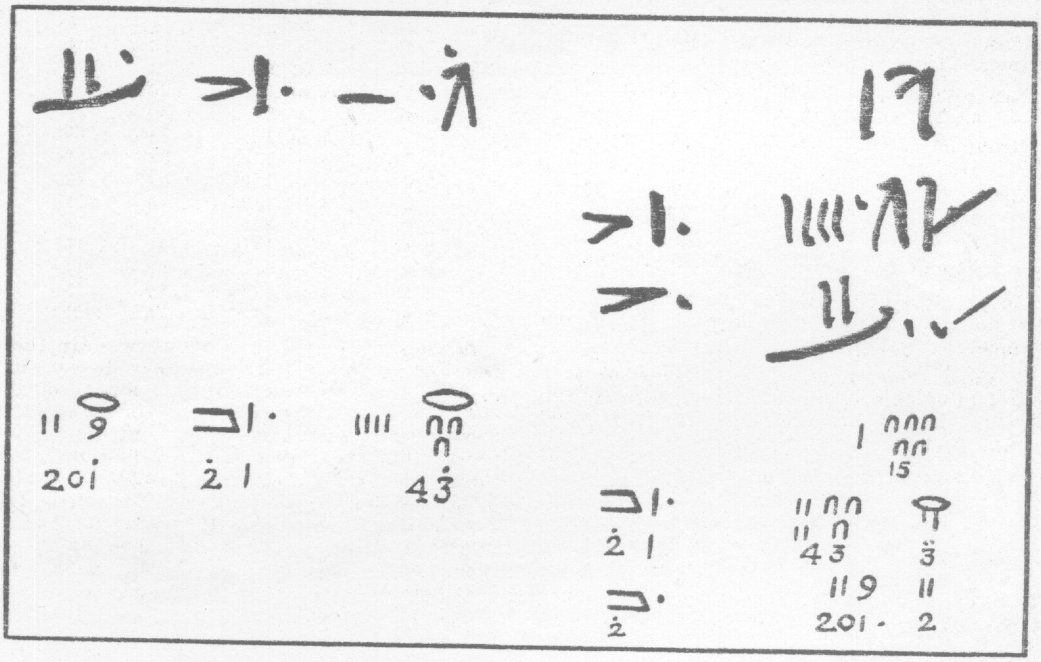

Figure 1

2 divided by 51.

	51	$\dot{3}4$	$1\dot{2}$	$\dot{1}02$	$[\dot{2}]$
\ $\dot{3}$	34		$1\dot{2}$		
\ 2	102		$\dot{2}$		

The extra 6 is written in red under the black 35, showing that he is using 6 × 35 as a common multiple—210. Then 1/35 applied to 210 is 6, and 2 times 6 is 12. Now 12 is partitioned into 7 + 5, which numbers he writes in black, under the red 1/30 and 1/42, indicating that 7 is 1/30 and 5 is 1/42 of the common multiple, 210. The check marks (shown by asterisks here), indicate on those lines that because 1 1/6 + 2/3 + 1/6 = 2, his answer, 2/35 = 1/30 + 1/42, is correct.

There are many other examples in the R.M.P. where the scribe uses such a common multiple. In problem 22 he adds the fractions 2/3, 1/6, 1/10, 1/30 by applying the denominators to the number 30, giving 20, 6, 3, and 1, showing 576 above. In problems 31, 36, 38, 37, 32, and 36 respectively, he uses the abundant numbers, 42, 56, 66, 288, 912, 1060, even though in some cases smaller numbers may have been suitable. It is this recognition and ready use of the multiple division properties of abundant numbers which gives the clue to the scribe's methods of evaluating 2 divided by the numbers of groups II and III, and of course group I, although he has a simpler method for the latter.

2.5 An abundant number is defined as a number the sum of whose divisors, including unity, is greater than the number itself, and is thus a number into which many other smaller numbers will divide exactly. The abundant numbers less than 100 are: **12, 18, 20,** 24, 30, 36, 40, 42, 48, 54, 56, 60, 66, 70, 72, 78, 80, 84, 88, 90, 96. The numbers 6 and 28, the sum of whose divisors is just equal to the numbers, are not abundant numbers but perfect numbers,[17] and are therefore not used by the scribe as I have indicated.

2.6 If the method is applied to multiples of 3, we can see how little different the setting out would appear, by again taking 2 divided by 51 as an illustration, for, as before, the whole 17 of them would be written in identical manner.

2 divided by 51

51	1/34 of 51 is 1 1/2, 1/102 of 51 is 1/2		
30	45	15	
	1		51
	*1/34		1 1/2
	*1/102		1/2

Thus he would use the abundant number 30 (chosen because 2 × 24 = 48 and is less than 51, and 2 × 36 = 72 is unnecessarily greater than 51), giving 30 × 51 = 1530. Then 1/51 applied to 1530 is 30, and 2 times 30 is 60. Now 60 can be partitioned as 45 + 15, which are 1/34 and 1/102 of 1530, so that 2/51 = 1/34 + 1/102. Not only may every division of group I be set down in exactly this fashion, *but the abundant number used is in every case the first one greater than* $1/2(n + 1)$, **where n is the** *particular division of 2 he is evaluating.* For 2 divided by 9, 15, and 21, the abundant number is 12; for 2 divided by 27 and 33 it is 18; for 2 divided by 39 it is 20; for 2 divided by 45 it is 24; for 2 divided by 51 and 57 it is 30; for 2 divided by 63 and 69 it is 36; for 2 divided by 75 it is 40; for 2 divided by 81 it is 42; for 2 divided by 87 and 93 it is 48; and for 2 divided by 99 it is 54.

One notes, of course, that the same answers are obtainable by using submultiples of the multipliers and the numbers obtained on partition, so that instead of 30, 45, 15, he could use, 6, 9, 3, or even 2, 3, 1, once he has determined the first set. Since in 2 divided by 35 the scribe wrote 6, 7, 5, when 30, 35, 25 are the numbers called for, we may surmise that the scribe would have written smaller submultiples.

3.1 *Group II. 2 divided by multiples of 5 and 7*

We can now set down the division of 2 by 5, 25, 35, 55, 65, 85, and 95 using the method of abundant numbers. The abundant numbers to be used here are 12, 18, 30, 30, 36, 48, and 60. The first one will serve as an illustration for this group.

2 divided by 5

5	1/3 of 5 is 1 2/3, 1/15 of 5 is 1/3	
12	20	4

Working out:	1	5
	*1/3	1 2/3
	*1/15	1/3

This means that since 5 × 12 = 60, then **1/5** applied to 60 is 12, and 2 times 12 is 24. Now 24 can be partitioned as 20 + 4, which are 1/3 and 1/15 of 60, so that 2/5 = 1/3 + 1/15. Of course in the R.M.P. the numbers 12, 20, 4 do not occur; they are written here to explain the method. In the case of 35 and 95 the scribe goes past the first suitable abundant numbers, because he can get simpler answers than they give. He passes 18 in the case of 2/35 because it leads to 1/18 + 1/630; passes 20 because it leads to 1/20 + 1/140; passes 24 because it leads to 1/24 + 1/105 + 1/168, and takes 30 because it leads to the simpler form, 1/30 + 1/42. Had he gone a step further to 36 he would have obtained 1/36 + 1/45 + 1/140 or 1/36 + 1/28 + 1/180, neither of which is as acceptable by his standards as the value he takes. Both 2/35 and 2/95 illustrate very well the method which I attribute to the author of the R.M.P. Thus in the latter he passes 48 because it leads to 1/95 + 1/4560 with a denominator over 1000; passes 54 because it leads to 1/54 + 1/513 + 1/1710 for the same reason; passes 56 because it leads to 1/56 + 1/532 + 1/760, and takes 60 because it leads to the simpler form 1/60 + 1/380 + 1/570. Had he gone a step further to 66 he would have obtained 1/66 + 1/285 + 1/418, which may be better if we knew exactly what his standards were—that is to say, does he prefer 1/60 to 1/66, although he has to take 1/570 instead of 1/418? We are, however, sure that had he gone further he would have fared worse, because 70 leads either to 1/70 + 1/190 + 1/665 or 1/70 + 1/175 + 1/950.

In the foregoing cases, as in all others, the scribe has chosen, with one or two rare and doubtful exceptions like the one above, the best answers possible according to his standards. In some cases he tries more numbers than for 95 above, until he finds the one he wants; in the case of 73 for example he tries six abundant numbers, 40, 42, 48, 54, 56, 60, before he finally takes 1/60 + 1/219 + 1/292 + 1/365, which is clearly the best offering, 66 and 70 giving worse values again. 2 divided by 73 is perhaps the best division of all to illustrate the method in its entirety. (See 4.4.)

3.2 All other unit fraction answers for multiples of 5 and 7 are obtained in exactly the same way, and we may summarize them as below more compactly, while showing all the relevant abundant numbers and their partitions.

2 divided by 5

1/5 applied to 60 is 12, and 2 × 12 = 24 = 20 + 4, i.e. 1/3 + 1/15 of 60.

2 divided by 25

1/25 applied to 450 is 18, and 2 × 18 = 36 = 30 + 6, i.e. 1/15 + 1/75 of 450.

2 divided by 35

1/35 applied to 1050 is 30, and 2 × 30 = 60 = 35 + 25, i.e. 1/30 + 1/42 of 1050.

2 divided by 55

1/55 applied to 1650 is 30, and 2 × 30 = 60 = 55 + 5, i.e. 1/30 + 1/330 of 1650.

2 divided by 65

1/65 applied to 2340 is 36, and 2 × 36 = 72 = 60 + 12, i.e. 1/39 + 1/195 of 2340.

2 divided by 85

1/85 applied to 4080 is 48, and 2 × 48 = 96 = 80 + 16, i.e. 1/51 + 1/255 of 4080.

2 divided by 95

1/95 applied to 5700 is 60, and 2 × 60 = 120 = 95 + 15 + 10, i.e. 1/60 + 1/380 + 1/570 of 5700.

3.3 Similarly for the multiples of 7 we have:

2 divided by 7

1/7 applied to 84 is 12, and 2 × 12 = 24 = 21 + 3, i.e. 1/4 + 1/28 of 84.

2 divided by 49

1/49 applied to 1764 is 36, and 2 × 36 = 72 = 63 + 9, i.e. 1/28 + 1/196 of 1764.

2 divided by 77

1/77 applied to 3080 is 40, and 2 × 40 = 80 = 70 + 10, i.e. 1/44 + 1/308 of 3080.

2 divided by 91

1/91 applied to 6370 is 70, and 2 × 70 = 140 = 91 + 49, i.e. 1/70 + 1/130 of 6370.

We observe that for 49 the scribe passes 30 because it leads to 1/42 + 1/98 + 1/147, three terms, when he can find a value with two. For 91, he passes 48, 54, 56, 60 and 66 in succession because they lead to 1/48 + 1/1456 + 1/2184, 1/54 + 1/351 + 1/1638, 1/56 + 1/392 + 1/637, 1/60 + 1/364 + 1/390, 1/66 + 1/154 + 1/3003 respectively, finally arriving at the relatively simple value 1/70 + 1/130 when he uses 70. This is another excellent illustration of the efficacy of the method.

4.1 *Group III, 2 divided by all the remaining primes*

These may be conveniently listed as for the members of group II. (Table I.)

4.2 The unit fractions for the primes above, which may appear to be arbitrary or haphazard are now seen to result from the operation of a definite procedure. Sometimes the seeking out of a partition of twice the abundant number is a laborious process, and the scribe may be forgiven if on occasion he misses a slightly simpler form, as in the case of 2 divided by 95 quoted earlier. The division of 2 by 13 is an interesting one. The first abundant number is 12, but the scribe will not take it, for it leads to the four fraction value, 1/12 + 1/26 + 1/52 + 1/78, and he looks for better. So [illegible] tries the abundant numbers, 18, 20 and 24, [illegible]d obtains in succession 1/9 + 1/26 + 1/234; 1/10 + 1/26 + 1/65; 1/8 + 1/52 + 1/104, the last of which he takes for his answer. It may well have been, therefore, that the scribe's preference was for 1/8 rather than 1/10, even though he had to take 1/104 instead of 1/65. That the scribe did not use a method equivalent to Eisenlohr's formula is evident, for in 2 divided by 13 he would thereby have obtained 1/7 + 1/91, which is a simpler answer than any others which his methods give him, and it does not come to his notice.

4.3 For each of 2 divided by 5, 7, 11, 17, 19, 23 the first abundant number is 12, and he uses it in all these cases. Other numbers can be found to serve equally well. For 5 he could have used 3, 6 or 9 as multipliers instead of 12. For 7, multipliers 4 or 8 would have given him the same answer. For 11, 6 would have served. For 17 the multiplier 9 would have given 1/9 + 1/153, and the multiplier 10 would have given 1/10 + 1/85 + 1/170, both of which

Table I

2 Divided by all the Remaining Primes

2 divided by,	the fraction,	applied to,	the Abundant Number	twice this,	its partitions,	which are the fractions,	of the number.
11	1/11	132	12	24	22+2	1/6+1/66	132
13	1/13	312	24	48	39+6+3	1/8+1/52+1/104	312
17	1/17	204	12	24	17+4+3	1/12+1/51+1/68	204
19	1/19	228	12	24	19+3+2	1/12+1/76+1/114	228
23	1/23	276	12	24	23+1	1/12+1/276	276
29	1/29	696	24	48	29+12+4+3	1/24+1/58+1/174+1/232	696
31	1/31	620	20	40	31+5+4	1/20+1/124+1/155	620
37	1/37	888	24	48	37+8+3	1/24+1/111+1/296	888
41	1/41	984	24	48	41+4+3	1/24+1/246+1/328	984
43	1/43	1806	42	84	43+21+14+6	1/42+1/86+1/129+1/301	1806
47	1/47	1410	30	60	47+10+3	1/30+1/141+1/470	1410
53	1/53	1590	30	60	53+5+2	1/30+1/318+1/795	1590
59	1/59	2124	36	72	59+9+4	1/36+1/236+1/531	2124
61	1/61	2440	40	80	61+10+5+4	1/40+1/244+1/488+1/610	2440
67	1/67	2680	40	80	67+8+5	1/40+1/335+1/536	2680
71	1/71	2840	40	80	71+5+4	1/40+1/568+1/710	2840
73	1/73	4380	60	120	73+20+15+12	1/60+1/219+1/292+1/365	4380
79	1/79	4740	60	120	79+20+15+6	1/60+1/237+1/316+1/790	4740
83	1/83	4980	60	120	83+15+12+10	1/60+1/332+1/415+1/498	4980
89	1/89	5340	60	120	89+15+10+6	1/60+1/356+1/534+1/890	5340
97	1/97	5432	56	112	97+8+7	1/56+1/679+1/776	5432
101	1/101	61206	606	1212	101+202+303+606	1/101+1/202+1/303+1/606	61206

the scribe ignores for 1/12 + 1/51 + 1/68 obtained from the multiplier 12. For 19 the scribe again ignores 1/10 + 1/190 from 10 as a multiplier, and takes 1/12 + 1/76 + 1/114 from 12 as a multiplier. For 23 the multiplier 12 gives 1/12 + 1/276, while the next abundant numbers, 18 and 20, lead to 1/18 + 1/46 + 1/138 + 1/414 and 1/20 + 1/46 + 1/92 + 1/230, neither of which are as good as the value he already has. It is now clear why the scribe's answers for 5, 7, 11 and 23, which appear to come as a result of Eisenlohr's formula, really come from a different technique and conform to it by accident. In fact, a complete table could easily be made using this formula, but the scribe is not seeking such answers. Indeed for all divisions after 43 the formula gives denominators greater than 1000, and so, for this reason alone, the scribe would not accept them.

4.4 For the remaining primes of group III, from 29 to 101, we can now make the following summary, which should be read in conjunction with the main table given in 4.1. (Table II.)

For 101 the multiplier must be an abundant number greater than 101, and furthermore must be a multiple of 101. None of 202, 303, 404 or 505 is an abundant number, but 606 is, and very simply produces the answer which the scribe gives, with denominators all less than 1000. It is clear, of course, once this is known, that the submultiple 6 would produce the answer.

4.5 I have mentioned earlier the possibility of errors on the part of the scribe, and Chace notes that the occasional mistakes are mostly numerical and merely accidental. Some are mistakes of copying by Ah-mose, and he may have added some details of his own. It is therefore worthy of remark that in his table of unit fractions we can find only a few doubtful cases, where he may have chosen better. Thus for 2/55 he chose 1/30 + 1/330, when 1/33 + 1/165 was available by partitioning 60 as 50 + 10 instead of 55 + 5. For 2/71 he chose 1/40 + 1/568 + 1/710 when by going a step farther to 42 he could have found 1/42 + 1/426 + 1/497, which is possibly a better value, if we are judging rightly. And for 2/95 he again chose 1/60 + 1/380 + 1/570, when 1/66 + 1/288 + 1/418 was available, by using the next higher abundant number, 66. Again we are not even certain that he was aware of both answers, and probably his choice was deliberate.

4.6 The example of 2 divided by 73 illustrates, perhaps best of all, the methods which I attribute to the author of the R.M.P. for the whole of this table. The smallest possible multiplier is 37, which gives 1/37 + 1/2701, which is rejected because of the denominator over 1000. Then the abundant multipliers in order give the following unit fraction values:

40 leads to 1/40 + 1/584 + 1/1460
42 leads to 1/42 + 1/441 + 1/1022 + 1/1533
48 leads to 1/48 + 1/219 + 1/876 + 1/1168
54 leads to 1/54 + 1/146 + 1/657 + 1/1971
56 leads to 1/56 + 1/146 + 1/584 + 1/1022
60 leads to 1/60 + 1/219 + 1/292 + 1/365
66 leads to 1/66 + 1/146 + 1/219 + 1/1606 + 1/4818

/511

TABLE II.

2 Divided by the Remaining Primes of Group III (29-101)

2 divided by	First abundant number	Scribe passes over	Scribe uses	Remarks
29	18	18, 20	24	18 gives only 3 terms, but the numbers are much bigger.
31	18	18	20	20 gives smaller numbers, and 24 leads to 4-term answers.
37	20	20	24	24 gives smaller numbers, and 30 leads to 4-term answers.
41	24	—	24	30 leads to 4-term answers.
43	24	24, 30, 36, 40	42	24 and 36 give only 3-term answers, but the numbers are bigger.
47	24	24	30	28 gives 1/28 + 1/188, but it is not an abundant number. 24 leads to numbers over 1000.
53	30	—	30	36 and 40 lead to 4-term answers, some over 1000.
59	30	30	36	30 and 40 give numbers over 1000, and 42 leads to a 5-term answer.
61	36	36	40	36, 42, 48 lead to 4-term answers with bigger numbers, or else to numbers over 1000.
67	36	36	40	36 gives numbers over 1000, and 42 gives much bigger numbers.
71	36	36	40	36 and 48 give numbers over 1000, and although 42 gives 1/42 + 1/426 + 1/497, he prefers the one he tried first.
73	40	40, 42, 48, 54, 56	60	All the numbers, as well as 66, lead to 4-digit denominators.
79	40	40, 42, 48, 54, 56	60	All the numbers give 4-digit numbers, one as large as 4424.
83	42	42, 48, 54, 56	60	42, 48, 54 give numbers bigger than 1000, and 56 gives bigger numbers than does 60.
89	48	48, 54, 56	60	48, 54, 56 lead to 4-digit denominators.
97	54	54	56	Both 54 and 60 give numbers over 1000.
101	54	—	606	All abundant numbers up to 102 give denominators over 1000.

Thus it is clear that the scribe persisted in his trials until he found, by using the abundant number 60 as a multiplier, the best value in unit fractions for 2 divided by 73, according to his standards.

Notes and References

(1) The numbers of the problems are those adopted in Chace, Bull, Manning, Archibald: The Rhind Mathematical Papyrus. 2 Vols. The Mathematical Association of America. Oberlin, Ohio, 1927. Hereafter referred to as R.M.P.

(2) R.M.P., p. 49. Free translation by Chace.

(3) The Egyptians had no sign for plus. Addition was indicated by simple juxtaposition, so that 1/5 + 1/45 would be written 1/5 1/45. The plus and equals signs used here and later are for the convenience of the reader.

(4) In the Mathematical Leather Roll, B.M.10250 in the British Museum, is found a table of equivalent fractions, in which one finds 1/5 = 1/10 + 1/10, 1/3 = 1/6 + 1/6, 2/3 = 1/3 + 1/3, 1/2 = 1/6 + 1/6 + 1/6, but such equivalents in calculations are rare.

(5) R.M.P., p. 15, footnote 1.

(6) Hultsch: Die Elemente der agyptischen Theilungsrechnung, 1895. Quoted in R.M.P., p. 14.

(7) T. Eric Peet: The Rhind Mathematical Papyrus, B.M.10057, 10058. London, 1923.

(8) Bibliography in R.M.P., by Archibald, pp. 142, 143.

(9) R.M.P., p. 15, footnote 2.

(10) Neugebauer, O.: Die Grundlagen der agyptischen Bruchrechnung. Berlin, 1926. The author's doctoral dissertation. See Archibald's bibliography, p. 189, R.M.P.

(11) R.M.P., p. 189.

(12) Problem number as given in R.M.P.

(13) The divisions of 2 by 5, 11, 17, 23 and 41.

(14) At first glance, 2 divided by 9 and 15 may appear to be set down in a slightly different manner. But in reality they are the same. The divisions by 9 and 15 concern such small numbers that the scribe omits one simple step in his proof.

(15) R.M.P., problem 33, p. 75. The scribe writes 2/3 of 42 = 28, and then 1/28 of 42 = 1 1/2, showing that the reciprocal of 2/3 is 1 1/2. This relation is also attested elsewhere.

(16) Heath, T. L.: The Thirteen Books of Euclid's Elements. 3 Vols. Cambridge, 19[illegible] In volume II, p. 293, Heath refers to t[illegible] as 'over-perfect' numbers.

(17) Euclid's Elements, Book IX, prop. 36, proves that if the sum of any number of the terms of the series, $1 + 2 + 2^2 + 2^3 + ---- + 2^{n-1}$ be prime, then this sum multiplied by the last term is a perfect number. See Heath, note 16, volume II, p. 421.

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