# THE ADDITION OF EGYPTIAN UNIT FRACTIONS

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A STUDENT of the ancient Egyptian exact sciences who studies the extant papyri soon becomes aware of what, at first glance, appears to be quite an 'oddity' in the solutions of the arithmetical problems. This 'oddity' is one of *emphasis*. For example, having due regard to the technique and methods available to the Egyptian scribe, a problem may be presented of some relative difficulty, and one finds explanatory matter, especially in the arithmetic, of the most elementary and simple kind set down in extensive detail, while the mechanism of a more abstruse and closely reasoned argument may be omitted altogether. It is as if the scribe was unaware of any inherent difficulty in it, and so he merely set down the answer. The arithmetical part of the following problem illustrates this.

Problem 70 of the Rhind Mathematical Papyrus (RMP)[1] proposes the question:

If 7 $\overline{2}$ $\overline{4}$ $\overline{8}$ *hekats*[2] of meal are made into 100 loaves, how much meal is there in each loaf, and what is the *pesu*[3] of the bread?

The *pesu* is a measure of the quality, concentration, or strength of the bread, as it is also of beer, and the higher the *pesu* number, the weaker the bread or beer, because less meal is used. The scribe shows how to divide 100 by 7 $\overline{2}$ $\overline{4}$ $\overline{8}$ to determine the *pesu*. He does this by his usual method for division, that is, to keep on multiplying his divisor (in this case 7 $\overline{2}$ $\overline{4}$ $\overline{8}$) until the dividend is reached (here it is 100). He obtains the answer 12 $\overline{\overline{3}}$ $\overline{42}$ $\overline{126}$, which is the required *pesu* number. This process is quite straightforward and simple enough, and he explains it in all detail. But the scribe, following his usual procedure elsewhere in the RMP, proceeds further by multiplying 12 $\overline{\overline{3}}$ $\overline{42}$ $\overline{126}$ by 7 $\overline{2}$ $\overline{4}$ $\overline{8}$ to obtain 100, the purpose being to prove that his answer is the correct one. This he must do using only unit fractions, that is, using fractions all of which have unity for their numerators,[4] with the solitary exception of the fraction $\frac{2}{3}$ for which he uses a special sign. In this operation, the scribe is confronted with the addition of a group of mixed numbers as follows:

the integers, 50 25 12 6 3 2 1 = 99,

[1] B.M. Pap. 10058, published Chace, Bull, Manning, Archibald: *The Rhind Mathematical Papyrus*. 2 vols. Math. Assoc. of America (Oberlin, Ohio, 1927).

[2] $\overline{2}$ $\overline{4}$ $\overline{8}$ is written instead of $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$ because all Egyptian fractions (except $\frac{2}{3}$) had unity for their numerators, and mere juxtaposition meant addition. $\frac{2}{3}$ is conventionally written $\overline{\overline{3}}$. *Ḥeḳats* may conveniently be translated as bushels.

[3] The relation *pesu* = (no. of loaves) ÷ (no. of *ḥeḳats*) shows that if, say, 100 loaves are made from 5 *ḥeḳats* of meal, the *pesu* of the bread would be 20, while if only 4 *ḥeḳats* of meal were used, the *pesu* would be 25, so that although of a higher *pesu* there is less meal in the bread. W. W. Struve uses the term 'Backverhältnis' in his translation of the Moscow Mathematical Papyrus (MMP).

[4] In the hieroglyphs, any integer became its reciprocal (or inverse) merely by writing ⬭ over it. In hieratic this became a dot.

and the sixteen unit fractions,

$$\overline{2}\ \overline{6}\ \overline{12}\ \overline{14}\ \overline{21}\ \overline{21}\ \overline{42}\ \overline{63}\ \overline{84}\ \overline{126}\ \overline{126}\ \overline{168}\ \overline{252}\ \overline{336}\ \overline{504}\ \overline{1008}.$$

This formidable array of fractions obviously adds up to 1, so as to give a complete total of 100, and, as far as we are able to judge, either the scribe adds them up in his head or he has close at hand previously prepared tables from which he makes hieratic jottings on some sort of ancient scribbling pad, writing down only his answer; for the RMP does not show us how the addition is made. Again in the Moscow Mathematical Papyrus, MMP,[1] the famous controversial problem 10 proposes to find the area of the curved surface of a 'basket',[2] shaped like a semi-cylinder, a hemisphere, or the half of an egg.[3] Only the directions for working with the given data are set down, and these rather briefly. How the scribe arrived at the answer of $\overline{\overline{3}}\ \overline{6}\ \overline{18}$, as the result of calculating $\overline{9}$ of 8 is not shown, nor indeed are we told how he found out that the result of multiplying $7\ \overline{9}$ by $4\ \overline{2}$ was 32. These are operations for which one might fairly expect some explanations.

In problem 9, the scribe of the MMP at once gives the answer to 11 divided by $1\ \overline{\overline{3}}\ \overline{6}$,[4] as if he worked the sum mentally, and in problem 17, confronted with the relatively more difficult operation of dividing $\overline{3}\ \overline{15}$ into 1, he merely sets down his answer as $2\ \overline{2}$, with no comment or explanation as to how he obtained it.

The 'oddity' of which we spoke comes more forcibly to our attention when we observe, in these same problems, that the scribe goes to the trouble of showing how he knows that 3 times 4 is 12, thus (RMP problem 26):

| | | | |
|---|---|---|---|
| If[5] | 1 | is | 3 |
| then | 2 | is | 6 |
| and | —4 | is | 12. |

Similarly, near the beginning of the RMP, he shows the complete working out for 4 times 7 is 28, thus:

| | | | |
|---|---|---|---|
| If | 1 | is | 7 |
| then | 2 | is | 14 |
| and | —4 | is | 28. |

He goes through similar processes in RMP problems 35 and 36, and in MMP problem 17, to establish the following simple results:

$$3\ \overline{3}\ \text{ times } 1 \text{ is } 3\ \overline{3}$$
$$3\ \overline{3}\ \overline{5}\ \text{ times } 1 \text{ is } 3\ \overline{3}\ \overline{5}$$
$$40\ \text{ times } 2\ \overline{2} \text{ is } 100,$$

any of which he could surely expect to be accepted without query.

[1] No. 4676, published by W. W. Struve, B. A. Turajeff, *Mathematischer Papyrus des Moskauer Museums der Schönen Künste, Quellen u. Studien*, Abt. A, Band 1 (Berlin, 1930).

[2] Struve has 'Körbe'.

[3] Peet (*JEA* 17, 154) regards the 'Körbe' as a semi-cylinder, Struve (see n. 1 above) as a hemisphere, while Neugebauer regards it as 'half-an-egg', *Vorgriechische Mathematik*. (Berlin, 1934). See also Van der Waerden, *Science Awakening*, 33–35 (Gröningen, 1954).

[4] The answer is correctly given as 6.

[5] The words are added for clarity, and the horizontal stroke to the left of 4 is a mark indicating the answer. In the papyrus this is sloped.

One is utterly unprepared, however, for the sheer simplicity of the operation of finding one-third of 3 (RMP problem 25). The scribe first finds two-thirds of 3 to be 2, and then halves 2 to get 1.

Again in problem 67 he finds one-third of unity as follows:

Do it thus:

If 1 is 1<br>
then $\overline{\overline{3}}$ is $\overline{\overline{3}}$<br>
and $\overline{3}$ is $\overline{3}$.

One can only be amazed at the trouble to which the scribe will go over things which appear to us so elementary; yet elsewhere, with operations on the addition of unit fractions of alarming proportions, he will produce immediate answers with the utmost nonchalance.

It is the purpose of this paper to determine how the scribes achieved, with such apparent facility, answers to relatively complex operations with fractions. Certainly the scribes did have some tables available, and the 26 entries on the addition of fractions in the Egyptian Mathematical Leather Roll (EMLR)[1] constituted probably one of many such prepared tables. The existence of tables is attested in the RMP itself, where, for example, the recto gives the results in unit fractions of the number 2 divided by all the odd numbers from 3 to 101.[2] This is quite an extensive table. Another table containing only 10 entries precedes problems 1 to 6.[3] In these problems the equalities listed in the table are specifically used. They consist of the results in unit fractions of the numbers from 1 to 10 divided by 10, one entry being, for example,

$$8 \div 10 = \overline{\overline{3}}\ \overline{10}\ \overline{30}.$$

Again, following problem 60, a table consisting of 15 entries gives a series of multiplications of fractions like

$$\overline{6} \text{ of } \overline{2} = \overline{12} \quad \text{and,}$$
$$\overline{7} \text{ of } \overline{\overline{3}} = \overline{14}\ \overline{42}.$$

Almost certainly a table for finding $\overline{\overline{3}}$ of any number,[4] integral or fractional, was available to the scribe, one which he must have regarded as of some importance, and it was probably of considerable length.

With straight out addition of integers the scribe clearly had no difficulties at all. Examples of this are quite numerous in the RMP and the MMP, and in upwards of 100 additions like the following, scarcely any scribal errors occur; but the evidence available does not enable us to say just how he did this, or whether he used any tables for straight out addition.

[1] The leather roll B.M. 10250 was acquired exactly 100 years ago by the British Museum together with the RMP, from the collection of A. H. Rhind. Roughly 10×20 in., it remained unrolled for over 60 years owing to its brittle condition. See Scott and Hall, *Brit. Mus. Quarterly* 2 (1927), 56; Glanville, 'Math. Leather Roll in the Br. Museum', *JEA* 13 (1927), 232 ff.; Vogel, *Arch. Gesch. Math. Naturwiss. Tech.* 2 (1929), 386; Neugebauer, *ZÄS* 64 (1929), 44; Gillings, *Australian Jr. Sci.* 24, no. 8 (1962), 339.

[2] See, for example, Gillings, 'The Division of the Odd Nos. 3 to 101 from the Recto of the RMP', *Austr. Jr. of Sci.* 18, no. 2 (1955), 43–49.

[3] Cf. Gillings, 'Problems 1–6 of the RMP', *The Maths. Teacher*, 55, no. 1 (1962), Washington, U.S.A.

[4] Cf. Gillings, 'The Egyptian $\frac{2}{3}$ Table for Fractions', *Aust. Jr. of Sci.* 22, no. 6 (1959), 247–50.

| *MMP (Pr. 11)* | *RMP (Pr. 31)* | *RMP (Pr. 70)* | *RMP (Pr. 30)* | *RMP (Pr. 79)* |
|---|---|---|---|---|
| 1 | 6 | 40 | 3621 | 7 |
| 8 | 21 | 80 | 1358 | 49 |
| 16 | 28 | 160 | 194 | 343 |
| — | 42 | 1280 | — | 2401 |
| 25 | — | 640 | 5173 | 16807 |
| | 97 | 320 | | — |
| | | — | | 19607 |
| | | 2520 | | |

For the addition of the smaller unit fractions,[1] the scribe shows that he knows at once by simple inspection that:

A. $\bar{2}\ \ \bar{2} = 1$ (26 examples from RMP and MMP)
B. $\bar{3}\ \ \bar{3} = \bar{\bar{3}}$ (8 " " " )
C. $\bar{\bar{3}}\ \ \bar{3} = 1$ (7 " " " )
D. $\bar{4}\ \ \bar{4} = \bar{2}$ (17 " " " )
E. $\bar{2}\ \ \bar{6} = \bar{\bar{3}}$ (10 " " " )
F. $\bar{3}\ \ \bar{6} = \bar{2}$ (9 " " " )
G. $\bar{6}\ \ \bar{6} = \bar{3}$ (7 " " " )
H. $\bar{2}\ \bar{3}\ \bar{6} = 1$ (3 " " " )

and many others, some of which involve the above.

The equalities which give us so much food for thought are those like the following, of which there is an abundance in both the RMP and the MMP. These and many others the scribe has summed immediately, writing down merely the answers.

$\bar{\bar{3}}\ \bar{\bar{3}}\ \bar{3}\ \bar{6}\ \bar{6} = 2$ (RMP Pr. 40)
$\bar{\bar{3}}\ \bar{6}\ \bar{9}\ \overline{18} = 1$ (RMP Pr. 67)
$\bar{2}\ \bar{5}\ \overline{10}\ \overline{10}\ \overline{10} = 1$ (RMP Pr. 35)
$\bar{\bar{3}}\ \bar{5}\ \overline{10}\ \overline{46}\ \overline{138}\ \overline{230} = 1$ (RMP Pr. 30)
$\bar{\bar{3}}\ \overline{10}\ \overline{10}\ \overline{15}\ \overline{30}\ \overline{30} = 1$ (RMP Pr. 30)
$\bar{\bar{3}}\ \bar{3}\ \bar{3}\ \bar{6}\ \bar{9}\ \bar{9}\ \overline{12}\ \overline{18}\ \overline{24}\ \overline{27}\ \overline{27}\ \overline{72}\ \overline{108} = 2$ (RMP Pr. 42)
$\bar{\bar{3}}\ \bar{6}\ \overline{10}\ \overline{30}\ \overline{30} = 1$ (MMP Pr. 17)

There are, of course, some very obvious simple groupings here, either in groups of two or three, or more, but there remain many for which the ordinary calculator must surely have required recourse to a set of tables in some form or other, unless he were a calculating prodigy. At this point we refer to the EMLR.

The 26 equalities in unit fractions of the EMLR divide themselves naturally into nine groups. There are 16 two-term equalities, 8 three-term equalities, and 2 four-term equalities. If we set them down in the order of these groups we at once become aware of an ordered sequence of unit fractions, of which the scribe must also have been aware. Further study leads one to the thought that, in the EMLR, we have only one (partial) table of many, which the scribe and his pupils must have constructed, and probably learnt by heart, in much the same way as a modern pupil learns his multiplication tables. There must have been many such tables, and we can perhaps think ourselves lucky that this one has come down to us in so legible a form.

[1] By 'smaller' fractions I mean fractions with smaller denominators. This is for brevity of reference.

## Re-arranged sequence of unit-fraction equalities of the EMLR

| Line | Group | Equality |
|---|---|---|
| 7 | Generator[1] (1, 1) | $\overline{3}\ \overline{3} = \overline{\overline{3}}$ |
| 5 | | $\overline{6}\ \overline{6} = \overline{3}$ |
| 4 | | $\overline{10}\ \overline{10} = \overline{5}$ |
| 11 | Generator (1, 2) | $\overline{9}\ \overline{18} = \overline{6}$ |
| 13 | | $\overline{12}\ \overline{24} = \overline{8}$ |
| 24 | | $\overline{15}\ \overline{30} = \overline{10}$ |
| 20 | | $\overline{18}\ \overline{36} = \overline{12}$ |
| 21 | | $\overline{21}\ \overline{42} = \overline{14}$ |
| 19 | | $\overline{24}\ \overline{48} = \overline{16}$ |
| 23 | | $\overline{30}\ \overline{60} = \overline{20}$ |
| 22 | | $\overline{45}\ \overline{90} = \overline{30}$ |
| 25 | | $\overline{48}\ \overline{96} = \overline{32}$ |
| 26 | | $\overline{96}\ \overline{192} = \overline{64}$ |
| 3 | Generator (1, 3) | $\overline{4}\ \overline{12} = \overline{3}$ |
| 2 | Generator (1, 4) | $\overline{5}\ \overline{20} = \overline{4}$ |
| 1 | | $\overline{10}\ \overline{40} = \overline{8}$ |
| 6 | Generator (1, 1, 1) | $\overline{6}\ \overline{6}\ \overline{6} = \overline{2}$ |
| 12 | Generator (1, 2, 4) | $\overline{7}\ \overline{14}\ \overline{28} = \overline{4}$ |
| 14 | Generator (2, 3, 6) | $\overline{14}\ \overline{21}\ \overline{42} = \overline{7}$ |
| 15 | | $\overline{18}\ \overline{27}\ \overline{54} = \overline{9}$ |
| 16 | | $\overline{22}\ \overline{33}\ \overline{66} = \overline{11}$ |
| 17 | | $\overline{26}\ \overline{39}\ \overline{78} = \overline{13}$ |
| 18 | | $\overline{30}\ \overline{45}\ \overline{90} = \overline{15}$ |
| 10 | Generator (1, 2, 6) | $\overline{25}\ \overline{50}\ \overline{150} = \overline{15}$ |
| 8 | Generator (3, 5, 15, 40) | $\overline{15}\ \overline{25}\ \overline{75}\ \overline{200} = \overline{8}$ |
| 9 | | $\overline{30}\ \overline{50}\ \overline{150}\ \overline{400} = \overline{16}$ |

**Generator (1, 1)**

| | |
|---|---|
| $\overline{2}\ \overline{2} = \overline{1}$ | |
| $\overline{3}\ \overline{3} = \overline{\overline{3}}$ | (l. 7) |
| $\overline{4}\ \overline{4} = \overline{2}$ | |
| $\overline{6}\ \overline{6} = \overline{3}$ | (l. 5) |
| $\overline{8}\ \overline{8} = \overline{4}$ | |
| $\overline{10}\ \overline{10} = \overline{5}$ | (l. 4) |
| $\overline{12}\ \overline{12} = \overline{6}$ | |

**Generator (1, 2)**

| | |
|---|---|
| $\overline{3}\ \overline{6} = \overline{2}$ | |
| $\overline{6}\ \overline{12} = \overline{4}$ | |
| $\overline{9}\ \overline{18} = \overline{6}$ | (l. 11) |
| $\overline{12}\ \overline{24} = \overline{8}$ | (l. 13) |
| $\overline{15}\ \overline{30} = \overline{10}$ | (l. 24) |
| $\overline{18}\ \overline{36} = \overline{12}$ | (l. 20) |
| $\overline{21}\ \overline{42} = \overline{14}$ | (l. 21) |
| $\overline{24}\ \overline{48} = \overline{16}$ | (l. 19) |
| $\overline{30}\ \overline{60} = \overline{20}$ | (l. 23) |
| $\overline{45}\ \overline{90} = \overline{30}$ | (l. 22) |
| $\overline{48}\ \overline{96} = \overline{32}$ | (l. 25) |
| $\overline{96}\ \overline{192} = \overline{64}$ | (l. 26) |

**Generator (1, 3)**

| | |
|---|---|
| $\overline{4}\ \overline{12} = \overline{3}$ | (l. 3) |
| $\overline{8}\ \overline{24} = \overline{6}$ | |
| $\overline{12}\ \overline{36} = \overline{9}$ | |
| $\overline{16}\ \overline{48} = \overline{12}$ | |
| $\overline{20}\ \overline{60} = \overline{15}$ | |

**Generator (1, 4)**

| | |
|---|---|
| $\overline{5}\ \overline{20} = \overline{4}$ | (l. 2) |
| $\overline{10}\ \overline{40} = \overline{8}$ | (l. 1) |
| $\overline{15}\ \overline{60} = \overline{12}$ | |
| $\overline{20}\ \overline{80} = \overline{16}$ | |
| $\overline{25}\ \overline{100} = \overline{20}$ | |

[1] I use this term to indicate the fundamental relation between the denominators of the unit fractions being added.

**Generator (1, 5)**

$\overline{6}\ \overline{30} = \overline{5}$

$\overline{12}\ \overline{60} = \overline{10}$

$\overline{18}\ \overline{90} = \overline{15}$

$\overline{24}\ \overline{120} = \overline{20}$

$\overline{30}\ \overline{150} = \overline{25}$

**Generator (1, 6)**

$\overline{7}\ \overline{42} = \overline{6}$

$\overline{14}\ \overline{84} = \overline{12}$

$\overline{21}\ \overline{126} = \overline{18}$

$\overline{28}\ \overline{168} = \overline{24}$

$\overline{35}\ \overline{210} = \overline{30}$

**Generator (2, 3)**

$\overline{10}\ \overline{15} = \overline{6}$

$\overline{20}\ \overline{30} = \overline{12}$

$\overline{30}\ \overline{45} = \overline{18}$

$\overline{40}\ \overline{60} = \overline{24}$

$\overline{50}\ \overline{75} = \overline{30}$

**Generator (2, 4) cf. (1, 2)**

$\overline{6}\ \overline{12} = \overline{4}$

$\overline{12}\ \overline{24} = \overline{8}$ (l. 13)

$\overline{18}\ \overline{36} = \overline{12}$ (l. 20)

$\overline{24}\ \overline{48} = \overline{16}$ (l. 19)

$\overline{30}\ \overline{60} = \overline{20}$ (l. 23)

**Generator (2, 5)**

$\overline{14}\ \overline{35} = \overline{10}$

$\overline{28}\ \overline{70} = \overline{20}$

$\overline{42}\ \overline{105} = \overline{30}$

$\overline{56}\ \overline{140} = \overline{40}$

$\overline{70}\ \overline{175} = \overline{50}$

**Generator (2, 6) cf. (1, 3)**

$\overline{4}\ \overline{12} = \overline{3}$ (l. 3)

$\overline{8}\ \overline{24} = \overline{6}$

$\overline{12}\ \overline{36} = \overline{9}$

$\overline{16}\ \overline{48} = \overline{12}$

$\overline{20}\ \overline{60} = \overline{15}$

**Generator (3, 4)**

$\overline{21}\ \overline{28} = \overline{12}$

$\overline{42}\ \overline{56} = \overline{24}$

$\overline{63}\ \overline{84} = \overline{36}$

$\overline{84}\ \overline{112}\ \ \overline{48}$

$\overline{105}\ \overline{140} = \overline{60}$

**Generator (3, 5)**

$\overline{24}\ \overline{40} = \overline{15}$

$\overline{48}\ \overline{80} = \overline{30}$

$\overline{72}\ \overline{120} = \overline{45}$

$\overline{96}\ \overline{160} = \overline{60}$

$\overline{120}\ \overline{200} = \overline{75}$

**Generator (3, 6) cf. (1, 2)**

$\overline{3}\ \overline{6} = \overline{2}$

$\overline{6}\ \overline{12} = \overline{4}$

$\overline{9}\ \overline{18} = \overline{6}$ (l. 11)

$\overline{12}\ \overline{24} = \overline{8}$ (l. 13)

$\overline{15}\ \overline{30} = \overline{10}$ (l. 24)

**Generator (4, 5)**

$\overline{36}\ \overline{45} = \overline{20}$

$\overline{72}\ \overline{90} = \overline{40}$

$\overline{108}\ \overline{135} = \overline{60}$

$\overline{144}\ \overline{180} = \overline{80}$

$\overline{180}\ \overline{225} = \overline{100}$

**Generator (4, 6) cf. (2, 3)**

$\overline{20}\ \overline{30} = \overline{12}$

$\overline{40}\ \overline{60} = \overline{24}$

$\overline{60}\ \overline{90} = \overline{36}$

$\overline{80}\ \overline{120} = \overline{48}$

$\overline{100}\ \overline{150} = \overline{60}$

**Generator (5, 6)**

$\overline{55}\ \overline{66} = \overline{30}$

$\overline{110}\ \overline{132} = \overline{60}$

$\overline{165}\ \overline{198} = \overline{90}$

$\overline{220}\ \overline{264} = \overline{120}$

$\overline{275}\ \overline{330} = \overline{150}$

**Generator (1, 1, 1)**

$\overline{3}\ \overline{3}\ \overline{3} = \overline{1}$

$\overline{6}\ \overline{6}\ \overline{6} = \overline{2}$ (l. 6)

$\overline{9}\ \overline{9}\ \overline{9} = \overline{3}$

$\overline{12}\ \overline{12}\ \overline{12} = \overline{4}$

$\overline{50}\ \overline{15}\ \overline{15} = \overline{5}$

**Generator (1, 2, 1)**

$\overline{5}\ \overline{10}\ \overline{5} = \overline{2}$

$\overline{10}\ \overline{20}\ \overline{10} = \overline{4}$

$\overline{15}\ \overline{30}\ \overline{15} = \overline{6}$

$\overline{20}\ \overline{40}\ \overline{20} = \overline{8}$

$\overline{25}\ \overline{50}\ \overline{25} = \overline{10}$

**Generator (1, 2, 2)**

$\overline{2}\ \overline{4}\ \overline{4} = \overline{1}$

$\overline{3}\ \overline{6}\ \overline{6} = \overline{\overline{3}}$

$\overline{4}\ \overline{8}\ \overline{8} = \overline{2}$

$\overline{6}\ \overline{12}\ \overline{12} = \overline{3}$

$\overline{8}\ \overline{16}\ \overline{16} = \overline{4}$

**Generator (1, 2, 3)**

$\overline{11}\ \overline{22}\ \overline{33} = \overline{6}$

$\overline{22}\ \overline{44}\ \overline{66} = \overline{12}$

$\overline{33}\ \overline{66}\ \overline{99} = \overline{18}$

$\overline{44}\ \overline{88}\ \overline{132} = \overline{24}$

$\overline{55}\ \overline{110}\ \overline{165} = \overline{30}$

**Generator (1, 2, 4)**

$\overline{7}\ \overline{14}\ \overline{28} = \overline{4}$ (l. 12)
$\overline{14}\ \overline{28}\ \overline{56} = \overline{8}$
$\overline{21}\ \overline{42}\ \overline{84} = \overline{12}$
$\overline{28}\ \overline{56}\ \overline{112} = \overline{16}$
$\overline{35}\ \overline{70}\ \overline{140} = \overline{20}$

**Generator (1, 2, 5)**

$\overline{17}\ \overline{34}\ \overline{85} = \overline{10}$
$\overline{34}\ \overline{68}\ \overline{170} = \overline{20}$
etc.

**Generator (1, 2, 6)**

$\overline{5}\ \overline{10}\ \overline{30} = \overline{3}$
$\overline{10}\ \overline{20}\ \overline{60} = \overline{6}$
$\overline{15}\ \overline{30}\ \overline{90} = \overline{9}$
$\overline{20}\ \overline{40}\ \overline{120} = \overline{12}$
$\overline{25}\ \overline{50}\ \overline{150} = \overline{15}$ (l. 10)

**Generator (1, 3, 3)**

$\overline{5}\ \overline{15}\ \overline{15} = \overline{3}$
$\overline{10}\ \overline{30}\ \overline{30} = \overline{6}$
etc.

**Generator (1, 3, 4)**

$\overline{19}\ \overline{57}\ \overline{76} = \overline{12}$
$\overline{38}\ \overline{114}\ \overline{152} = \overline{24}$
etc.

**Generator (1, 3, 5)**

$\overline{23}\ \overline{69}\ \overline{115} = \overline{15}$
$\overline{46}\ \overline{138}\ \overline{230} = \overline{30}$
etc.

**Generator (1, 3, 6)**

$\overline{3}\ \overline{9}\ \overline{18} = \overline{2}$
$\overline{6}\ \overline{18}\ \overline{36} = \overline{4}$
$\overline{9}\ \overline{27}\ \overline{54} = \overline{6}$
$\overline{12}\ \overline{36}\ \overline{72} = \overline{8}$
$\overline{15}\ \overline{45}\ \overline{90} = \overline{10}$

**Generator (1, 4, 4)**

$\overline{6}\ \overline{24}\ \overline{24} = \overline{4}$
$\overline{12}\ \overline{48}\ \overline{48} = \overline{8}$
etc.

**Generator (1, 4, 5)**

$\overline{29}\ \overline{116}\ \overline{145} = \overline{20}$
$\overline{58}\ \overline{232}\ \overline{290} = \overline{40}$
etc.

**Generator (1, 4, 6)**

$\overline{17}\ \overline{68}\ \overline{102} = \overline{12}$
$\overline{34}\ \overline{136}\ \overline{204} = \overline{24}$
etc.

**Generator (1, 5, 5)**

$\overline{7}\ \overline{35}\ \overline{35} = \overline{5}$
$\overline{14}\ \overline{70}\ \overline{70} = \overline{10}$
etc.

**Generator (1, 5, 6)**

$\overline{41}\ \overline{205}\ \overline{246} = \overline{30}$
$\overline{82}\ \overline{410}\ \overline{492} = \overline{60}$
etc.

**Generator (1, 6, 6)**

$\overline{4}\ \overline{24}\ \overline{24} = \overline{3}$
$\overline{8}\ \overline{48}\ \overline{48} = \overline{6}$
etc.

**Generator (2, 3, 3)**

$\overline{14}\ \overline{21}\ \overline{21} = \overline{6}$
$\overline{28}\ \overline{42}\ \overline{42} = \overline{12}$
etc.

**Generator (2, 3, 4)**

$\overline{26}\ \overline{39}\ \overline{52} = \overline{12}$
$\overline{52}\ \overline{78}\ \overline{104} = \overline{24}$
etc.

**Generator (2, 3, 5)**

$\overline{62}\ \overline{93}\ \overline{155} = \overline{30}$
$\overline{124}\ \overline{186}\ \overline{310} = \overline{60}$
etc.

**Generator (2, 3, 6)**

$\overline{2}\ \overline{3}\ \overline{6} = \overline{1}$
$\overline{4}\ \overline{6}\ \overline{12} = \overline{2}$
$\overline{6}\ \overline{9}\ \overline{18} = \overline{3}$
$\overline{8}\ \overline{12}\ \overline{24} = \overline{4}$
$\overline{10}\ \overline{15}\ \overline{30} = \overline{5}$
$\overline{12}\ \overline{18}\ \overline{36} = \overline{6}$
$\overline{14}\ \overline{21}\ \overline{42} = \overline{7}$ (l. 14)
$\overline{16}\ \overline{24}\ \overline{48} = \overline{8}$
$\overline{18}\ \overline{27}\ \overline{54} = \overline{9}$ (l. 15)
$\overline{20}\ \overline{30}\ \overline{60} = \overline{10}$
$\overline{22}\ \overline{33}\ \overline{66} = \overline{11}$ (l. 16)
$\overline{24}\ \overline{36}\ \overline{72} = \overline{12}$
$\overline{26}\ \overline{39}\ \overline{78} = \overline{13}$ (l. 17)
$\overline{28}\ \overline{42}\ \overline{84} = \overline{14}$
$\overline{30}\ \overline{45}\ \overline{90} = \overline{15}$ (l. 18)

**Generator (2, 4, 4) cf. (1, 2, 2)**

$\overline{4}\ \overline{8}\ \overline{8} = \overline{2}$
$\overline{6}\ \overline{12}\ \overline{12} = \overline{3}$
etc.

**Generator (2, 4, 5)**

$\overline{38}\ \overline{76}\ \overline{95} = \overline{20}$
$\overline{76}\ \overline{152}\ \overline{190} = \overline{40}$
etc.

**Generator (2, 4, 6) cf. (1, 2, 3)**

$$\overline{22}\ \overline{44}\ \overline{66} = \overline{12}$$
$$\overline{44}\ \overline{88}\ \overline{132} = \overline{24}$$
$$\overline{66}\ \overline{132}\ \overline{198} = \overline{36}$$
$$\overline{88}\ \overline{176}\ \overline{264} = \overline{48}$$
$$\overline{110}\ \overline{220}\ \overline{330} = \overline{60}$$

**Generator (2, 5, 5)**

$$\overline{18}\ \overline{45}\ \overline{45} = \overline{10}$$
$$\overline{36}\ \overline{90}\ \overline{90} = \overline{20}$$

etc.

**Generator (2, 5, 6)**

$$\overline{26}\ \overline{65}\ \overline{78} = \overline{15}$$
$$\overline{52}\ \overline{130}\ \overline{156} = \overline{30}$$

etc.

**Generator (3, 4, 4)**

$$\overline{15}\ \overline{20}\ \overline{20} = \overline{6}$$
$$\overline{30}\ \overline{40}\ \overline{40} = \overline{12}$$

etc.

**Generator (3, 4, 5)**

$$\overline{141}\ \overline{188}\ \overline{235} = \overline{60}$$
$$\overline{282}\ \overline{376}\ \overline{470} = \overline{120}$$

**Generator (3, 4, 6)**

$$\overline{9}\ \overline{12}\ \overline{18} = \overline{4}$$
$$\overline{18}\ \overline{24}\ \overline{36} = \overline{8}$$
$$\overline{27}\ \overline{36}\ \overline{54} = \overline{12}$$
$$\overline{36}\ \overline{48}\ \overline{72} = \overline{16}$$
$$\overline{45}\ \overline{60}\ \overline{90} = \overline{20}$$

etc.

**Generator (3, 5, 5)**

$$\overline{33}\ \overline{55}\ \overline{55} = \overline{15}$$
$$\overline{66}\ \overline{110}\ \overline{110} = \overline{30}$$

etc.

**Generator (3, 5, 6)**

$$\overline{21}\ \overline{35}\ \overline{42} = \overline{10}$$
$$\overline{42}\ \overline{70}\ \overline{84} = \overline{20}$$

etc.

**Generator (3, 6, 6) cf. (1, 2, 2)**

$$\overline{12}\ \overline{24}\ \overline{24} = \overline{6}$$
$$\overline{24}\ \overline{48}\ \overline{48} = \overline{12}$$

etc.

**Generator (4, 5, 6)**

$$\overline{148}\ \overline{185}\ \overline{222} = \overline{60}$$
$$\overline{296}\ \overline{370}\ \overline{444} = \overline{120}$$

etc.

**Generator (4, 6, 6) cf. (2, 3, 3)**

$$\overline{28}\ \overline{42}\ \overline{42} = \overline{12}$$
$$\overline{56}\ \overline{84}\ \overline{84} = \overline{24}$$

etc.

**Generator (1, 2, 3, 4)**

$$\overline{25}\ \overline{50}\ \overline{75}\ \overline{100} = \overline{12}$$
$$\overline{50}\ \overline{100}\ \overline{150}\ \overline{200} = \overline{24}$$

etc.

**Generator (1, 2, 3, 5)**

$$\overline{61}\ \overline{122}\ \overline{183}\ \overline{305} = \overline{30}$$
$$\overline{122}\ \overline{244}\ \overline{366}\ \overline{610} = \overline{60}$$

etc.

**Generator (1, 2, 3, 6)**

$$\overline{2}\ \overline{4}\ \overline{6}\ \overline{12} = \overline{1}$$
$$\overline{4}\ \overline{8}\ \overline{12}\ \overline{24} = \overline{2}$$

etc.

**Generator (1, 2, 4, 5)**

$$\overline{39}\ \overline{78}\ \overline{156}\ \overline{195} = \overline{20}$$
$$\overline{78}\ \overline{156}\ \overline{312}\ \overline{390} = \overline{40}$$

etc.

**Generator (1, 2, 4, 6)**

$$\overline{23}\ \overline{46}\ \overline{92}\ \overline{138} = \overline{12}$$
$$\overline{46}\ \overline{92}\ \overline{184}\ \overline{276} = \overline{24}$$

etc.

**Generator (1, 2, 5, 6)**

$$\overline{28}\ \overline{56}\ \overline{140}\ \overline{168} = \overline{15}$$
$$\overline{56}\ \overline{112}\ \overline{280}\ \overline{336} = \overline{30}$$

etc.

**Generator (1, 3, 4, 5)**

$$\overline{107}\ \overline{321}\ \overline{428}\ \overline{535} = \overline{60}$$
$$\overline{214}\ \overline{642}\ \overline{856}\ \overline{1070} = \overline{120}$$

etc.

**Generator (1, 3, 4, 6)**

$$\overline{7}\ \overline{21}\ \overline{28}\ \overline{42} = \overline{4}$$
$$\overline{14}\ \overline{42}\ \overline{56}\ \overline{84} = \overline{8}$$

etc.

**Generator (1, 3, 5, 6)**

$$\overline{17}\ \overline{51}\ \overline{85}\ \overline{102} = \overline{10}$$
$$\overline{34}\ \overline{102}\ \overline{170}\ \overline{204} = \overline{20}$$

etc.

**Generator (1, 4, 5, 6)**

$$\overline{97}\ \ \overline{388}\ \ \overline{485}\ \ \overline{582} = \overline{60}$$
$$\overline{194}\ \ \overline{776}\ \ \overline{970}\ \ \overline{1164} = \overline{120}$$

etc.

**Generator (2, 3, 5, 6)**

$$\overline{12}\ \ \overline{18}\ \ \overline{30}\ \ \overline{36} = \overline{5}$$
$$\overline{24}\ \ \overline{36}\ \ \overline{60}\ \ \overline{72} = \overline{10}$$

etc.

**Generator (2, 3, 4, 5)**

$$\overline{154}\ \ \overline{231}\ \ \overline{308}\ \ \overline{385} = \overline{60}$$
$$\overline{308}\ \ \overline{462}\ \ \overline{616}\ \ \overline{770} = \overline{120}$$

etc.

**Generator (2, 4, 5, 6)**

$$\overline{134}\ \ \overline{268}\ \ \overline{335}\ \ \overline{402} = \overline{60}$$
$$\overline{268}\ \ \overline{536}\ \ \overline{670}\ \ \overline{804} = \overline{120}$$

etc.

**Generator (2, 3, 4, 6)**

$$\overline{10}\ \ \overline{15}\ \ \overline{20}\ \ \overline{30} = \overline{4}$$
$$\overline{20}\ \ \overline{30}\ \ \overline{40}\ \ \overline{60} = \overline{8}$$

etc.

**Generator (3, 4, 5, 6)**

$$\overline{57}\ \ \overline{76}\ \ \overline{95}\ \ \overline{114} = \overline{20}$$
$$\overline{114}\ \ \overline{152}\ \ \overline{190}\ \ \overline{228} = \overline{40}$$

etc.

The two-termed equalities can now be seen to be related so that in the second group, one term is always double the other, in the third group one term is always three times the other, and in the fourth group one term is four times the other. In the three-termed equalities, the ratios in the groups of the three terms are (1, 1, 1), (1, 2, 4), (2, 3, 6), and (1, 2, 6) while in the four-termed equalities the ratios of the terms are (3, 5, 15, 40).

It is obvious that no wholly complete table of such unit fractions is possible, because their number must be infinite. We can, however, make a pretty useful and comprehensive set of tables if we limit ourselves to the numbers 1 to 6 and consider all the combinations of unit fractions, two, three, and four at a time. If we do this we will see that, in the EMLR, the scribe has included a fairly comprehensive sampling, together with one or two others.

The reason for extending the members of the groups (1, 2) and (2, 3, 6) a little more than the others is because they contain so many of the scribe's entries in the EMLR. From the sequence of the five lines numbered 14, 15, 16, 17, 18 in the (2, 3, 6) group, we may conclude that the scribe composed his tables in some manner similar to the one adopted here, namely by simple multiplication (or indeed addition) of the known equalities, to obtain a new equality. And we may further note that the scribal error of line 17, where the scribe had written,

$$\overline{28}\ \ \overline{49}\ \ \overline{196} = \overline{13},$$

is more likely to have been,

$$\overline{26}\ \ \overline{39}\ \ \overline{78} = \overline{13},$$

which correction we have incorporated in the foregoing tables, rather than the one suggested by Glanville,[1] which was

$$\overline{28}\ \ \overline{49}\ \ \overline{98}\ \ \overline{196} = \overline{14}.$$

In setting down these tables for the addition of unit Egyptian fractions in this particular way, I am sure we are merely repeating, in essence, what the Egyptian scribes had already done three or four millennia ago, and in the summations which follow, we use the tables in much the same way in which, I feel, the scribe himself must have used them.

[1] *JEA* 13, 237.

## Addition of fractions

*Pr. 40*

$$\overline{\overline{3}}\ \ \overline{\overline{3}}\ \ \overline{3}\ \ \overline{6}\ \ \overline{6}.$$

We could bracket the last three terms $(\overline{3}\ \overline{6}\ \overline{6}) = \overline{\overline{3}}$ from the generator (1, 2, 2) and get $\overline{\overline{3}}\ \overline{\overline{3}}\ \overline{\overline{3}}$ and proceed from there, but we bracket $(\overline{\overline{3}}\ \overline{\overline{3}})$ and $(\overline{6}\ \overline{6})$ getting $1\ \overline{3}$ and $\overline{3}$ from the generator (1, 1), and we have,

$$1\ \ \overline{3}\ \ \overline{3}\ \ \overline{3}.$$

From the generator (1, 1, 1) we find $(\overline{3}\ \overline{3}\ \overline{3}) = 1$ and we have,

$$1\ \ 1\ \ =\ \ 2.$$

*Pr. 67*

$$\overline{\overline{3}}\ \ \overline{6}\ \ \overline{9}\ \ \overline{18}.$$

By bracketing the terms $(\overline{6}\ \overline{9}\ \overline{18}) = \overline{3}$ from the generator (2, 3, 6) we have,

$\overline{\overline{3}}\ \ \overline{3}\ \ =\ \ 1$ from the equality (C).

*Pr. 35*

$$\overline{2}\ \ \overline{5}\ \ \overline{10}\ \ \overline{10}\ \ \overline{10}.$$

No grouping of three terms presents itself. We must put $(\overline{10}\ \overline{10}) = \overline{5}$ from the generator (1, 1), and we now have,

$$\overline{2}\ \ \overline{5}\ \ \overline{5}\ \ \overline{10}.$$

The only possible grouping here is $(\overline{5}\ \overline{5})$ which from the Recto $(2 \div 5)$ of the RMP equals $\overline{3}\ \overline{15}$, so that we have,

$$\overline{2}\ \ \overline{3}\ \ \overline{10}\ \ \overline{15}.$$

We may now group $(\overline{10}\ \overline{15}) = \overline{6}$ from the generator (2, 3) and obtain,

$\overline{2}\ \ \overline{3}\ \ \overline{6}\ \ =\ \ 1$, from the generator (2, 3, 6).

*Pr. 30*

$$\overline{\overline{3}}\ \ \overline{5}\ \ \overline{10}\ \ \overline{46}\ \ \overline{138}\ \ \overline{230}.$$

The last three terms may be grouped using the generator (1, 3, 5), to give $(\overline{46}\ \overline{138}\ \overline{230}) = \overline{30}$, and we have,

$$\overline{\overline{3}}\ \ \overline{5}\ \ \overline{10}\ \ \overline{30}.$$

Again we put together the last three terms $(\overline{5}\ \overline{10}\ \overline{30}) = \overline{3}$ using the generator (1, 2, 6) so that we get,

$\overline{\overline{3}}\ \ \overline{3}\ \ =\ \ 1$, from the equality (C).

*Pr. 30*

$$\overline{\overline{3}}\ \ \overline{10}\ \ \overline{10}\ \ \overline{15}\ \ \overline{30}\ \ \overline{30}.$$

We first bracket the three terms $(\overline{10}\ \overline{15}\ \overline{30}) = \overline{5}$ from the generator (2, 3, 6) so that we have

$$\overline{\overline{3}}\ \ \overline{10}\ \ \overline{5}\ \ \overline{30}.$$

Again by bracketing the last three terms $(\overline{5}\ \overline{10}\ \overline{30}) = \overline{3}$, from the generator (1, 2, 6), we get,

$\overline{\overline{3}}\ \ \overline{3}\ \ =\ \ 1$ from the equality (C).

*Pr. 42*

$$\overline{\overline{3}}\ \ \overline{3}\ \ \overline{3}\ \ \overline{6}\ \ \overline{9}\ \ \overline{9}\ \ \overline{12}\ \ \overline{18}\ \ \overline{24}\ \ \overline{27}\ \ \overline{27}\ \ \overline{72}\ \ \overline{108}.$$

Excepting the obvious groupings of $(\overline{\overline{3}}\ \overline{3})$ and $(\overline{3}\ \overline{6})$, no other immediate groupings involving the use of the tables lead to a simple solution. We therefore resort to the Recto table applied to $(\overline{9}\ \overline{9})$ and $(\overline{27}\ \overline{27})$ and we have,

$(\overline{\overline{3}}\ \ \overline{3})\ \ (\overline{3}\ \ \overline{6})\ \ (\overline{9}\ \ \overline{9})\ \ \overline{12}\ \ \overline{18}\ \ \overline{24}\ \ (\overline{27}\ \ \overline{27})\ \ \overline{72}\ \ \overline{108}$,

$=\ \ 1\ \ \overline{2}\ \ (\overline{6}\ \ \overline{18})\ \ \overline{12}\ \ \overline{18}\ \ \overline{24}\ \ (\overline{18}\ \ \overline{54})\ \ \overline{72}\ \ \overline{108}$, and re-grouping,

$=\ \ 1\ \ \overline{2}\ \ \overline{6}\ \ (\overline{18}\ \ \overline{18}\ \ \overline{18})\ \ (\overline{12}\ \ \overline{24})\ \ \overline{72}\ \ (\overline{54}\ \ \overline{108})$,

using the generators (1, 1, 1), (1, 2), and (1, 2) again, we get,

$$= 1 \quad \overline{2} \quad \overline{6} \quad \overline{6} \quad \overline{8} \quad \overline{72} \quad \overline{36}.$$

Now we group ($\overline{6}$ $\overline{6}$) and ($\overline{36}$ $\overline{72}$) using generators (1, 1) and (1, 2).
$= 1\ \overline{2}\ \overline{3}\ \overline{8}\ \overline{24}$, and then grouping ($\overline{8}$ $\overline{24}$) using (1, 3),
$= 1\ \overline{2}\ \overline{3}\ \overline{6}$, and the final 3 fractions, being a fundamental generator giving $\overline{2}\ \overline{3}\ \overline{6} = 1$, we have,
$= 1\ 1$
$= 2.$

*Pr. 17 MMP.*

$$\overline{\overline{3}} \quad \overline{6} \quad \overline{10} \quad \overline{30} \quad \overline{30}.$$

The grouping of the last three terms giving ($\overline{10}$ $\overline{30}$ $\overline{30}$) = $\overline{6}$ is possible, but it seems more likely that the scribe would use the generator (1, 5) on the 2nd and 4th terms, giving ($\overline{6}$ $\overline{30}$) = $\overline{5}$ so that the line now reads,

$$\overline{\overline{3}} \quad \overline{5} \quad \overline{10} \quad \overline{30}.$$

Now by grouping the last three terms, and from the generator (1, 2, 6) we will have,

$$\overline{\overline{3}} \quad \overline{3} \quad = \quad 1 \text{ from the equality (C).}$$

And now we return to the original problem of the summation of the sixteen fractions of problem 70 of the RMP.

$$\overline{2} \quad \overline{6} \quad \overline{14} \quad \overline{21} \quad \overline{21} \quad \overline{42} \quad \overline{63} \quad \overline{84} \quad \overline{126} \quad \overline{126} \quad \overline{168} \quad \overline{252} \quad \overline{336} \quad \overline{504} \quad \overline{1008}.$$

Starting with the higher numbers we first note that 336, 504, and 1008 are each multiples of 168, so that by using the generator (1, 2, 3, 6), we can obtain ($\overline{168}$ $\overline{336}$ $\overline{504}$ $\overline{1008}$) = $\overline{84}$.
The line now reads,

$$\overline{2} \quad \overline{6} \quad \overline{12} \quad \overline{14} \quad \overline{21} \quad \overline{21} \quad \overline{42} \quad \overline{63} \quad \overline{84} \quad \overline{84} \quad \overline{126} \quad \overline{126} \quad \overline{252}.$$

We might be tempted to group ($\overline{84}$ $\overline{84}$) = $\overline{42}$ and ($\overline{126}$ $\overline{126}$) = $\overline{63}$, but again we note that 84, 126, and 252 are multiples of 42, so that using the same generator (1, 2, 3, 6) we get ($\overline{42}$ $\overline{84}$ $\overline{126}$ $\overline{252}$) = $\overline{21}$.
Now it reads,

$$\overline{2} \quad \overline{6} \quad \overline{12} \quad \overline{14} \quad \overline{21} \quad \overline{21} \quad \overline{21} \quad \overline{63} \quad \overline{84} \quad \overline{126}.$$

We now bracket ($\overline{63}$ $\overline{84}$ $\overline{126}$) = $\overline{28}$, for these numbers are multiples of 21, and also ($\overline{21}$ $\overline{21}$ $\overline{21}$) = $\overline{7}$, and using the generators (3, 4, 6) and (1, 1, 1) we have,

$$\overline{2} \quad \overline{6} \quad \overline{12} \quad \overline{14} \quad \overline{7} \quad \overline{28}.$$

Bracketing ($\overline{7}$ $\overline{14}$ $\overline{28}$) = $\overline{4}$ and ($\overline{6}$ $\overline{12}$) = $\overline{4}$ from the generators (1, 2, 4) and (1, 2), we reduce to

$$\overline{2} \quad \overline{4} \quad \overline{4} \quad \text{which is unity from the generator (1, 2, 2).}$$

One must remark that the groupings suggested in the foregoing examples are only some possible sequences in a large number of others; a scribe skilled in this technique might perform the summation in a shorter or more elegant fashion. On the other hand, a scribe not so skilled, or one who perhaps did not have his reference tables handy, could use another method which is well attested in the RMP. Thus in problem 22 it is required to show that

$$\overline{\overline{3}} \quad \overline{5} \quad \overline{10} \quad \overline{30} \quad = \quad 1.$$

Now by grouping the last three fractions, and using the generator (1, 2, 6), the scribe can easily obtain ($\overline{5}$ $\overline{10}$ $\overline{30}$) = $\overline{3}$, so that he could obtain $\overline{\overline{3}}$ $\overline{3}$ = 1. But he does not do this.

Instead he directs the student to apply the four fractions separately to the number 30, which he then uses as a sort of 'least common multiple'. By taking in turn $\overline{\overline{3}}$, $\overline{5}$, $\overline{10}$, and $\overline{30}$ of 30 he gets 20, 6, 3, 1 which total 30, thus proving that

$$\overline{\overline{3}} \quad \overline{5} \quad \overline{10} \quad \overline{30} \quad = \quad 1.$$

This method of adding fractions is employed by the scribe of the RMP in his problems on completion (problems 21, 22, 23) clearly for purposes of teaching the technique, and he sometimes has occasion to use it in his subsequent work. But in by far the majority of cases, the scribe is able to sum up to twenty and more fractions with quite large denominators without the use of this, what we may call, LCM method.

In problem 32 of the RMP the scribe has to sum the fractions

$$\overline{12} \quad \overline{18} \quad \overline{24} \quad \overline{36} \quad \overline{48} \quad \overline{114} \quad \overline{228} \quad \overline{342} \quad \overline{456} \quad \overline{684} \quad \overline{912} \text{ to give } \overline{4}.$$

Presumably because his tables for the addition of unit fractions were not close at hand, or for some other reason, perhaps a pedagogical one, he uses the highest number in the set, 912, as a common multiple (it is not, of course, what we call the LCM, which is 2736), and by dividing the eleven numbers of the fractions into 912, he obtains the series

$$76 \quad 50\ \overline{\overline{3}} \quad 38 \quad 25\ \overline{3} \quad 19 \quad 8 \quad 4 \quad 2\ \overline{3} \quad 2 \quad 1\overline{3} \quad 1,$$

which he says together make 228, or $\overline{4}$ of 912. The scribe does not show any of these eleven divisions, but 'oddly' enough he does show that $\overline{4}$ of 912 is 228, as his usual check, as follows:

| | |
|---|---|
| 1 | 912 |
| $\overline{2}$ | 456 |
| —$\overline{4}$ | 228 |

In my own trial for the addition of these fractions I found this method very much more laborious than grouping the fractions and using tables. For the eleven divisions take some time as well as space, while the obvious groupings using the generator (1, 2) almost demand to be used; so that in a few lines we obtain,

$$\overline{12} \quad (\overline{18} \quad \overline{36}) \quad (\overline{24} \quad \overline{48}) \quad (\overline{114} \quad \overline{228}) \quad (\overline{342} \quad \overline{684}) \quad (\overline{456} \quad \overline{912})$$
$$= \overline{12} \qquad \overline{12} \qquad \overline{16} \qquad \overline{76} \qquad \overline{228} \qquad \overline{304}.$$

Since the last three fractions belong to the (1, 3, 4) generating group, the fractions $= \overline{12} \quad \overline{12} \quad \overline{16} \quad \overline{48}$, and since $(\overline{16}\ \overline{48}) = \overline{12}$ from the generator (1, 3), $= \overline{12} \quad \overline{12} \quad \overline{12}$ which is $\overline{4}$.