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PROBLEMS BUREAU

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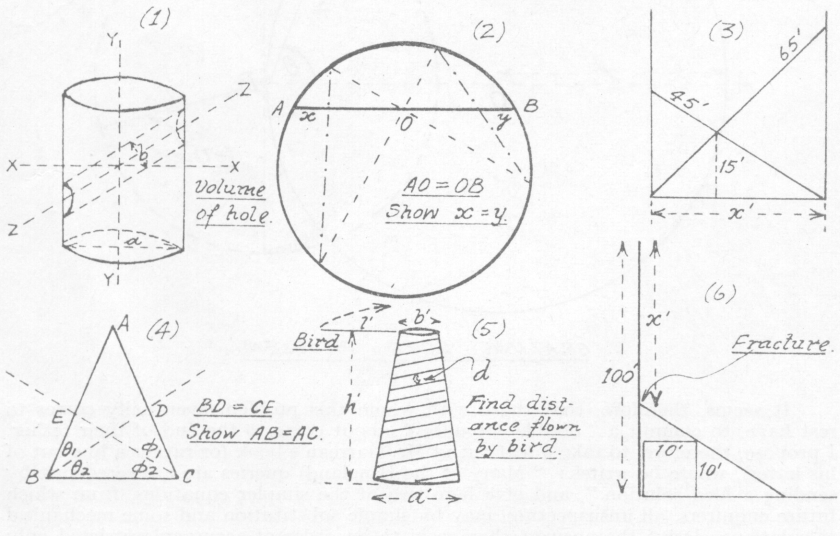
One of the first problems received by the bureau when it started in 1939 was that of the *Grazing Goat*, by which odd name it has always been known, although it has variously been presented as the grazing sheep, cow, horse, and even a donkey, the latter indeed by J. Pedoe in *The Mathematical Gazette* of *October* 1940, *Mathematical Note* No. 1477. It stands out head and shoulders above any other problem presented to us for solution in the matter of frequency, the next closest of the " Hardy Annuals ", using the nomenclature I adopt for handy reference, being

1. Volume of a hole through a cylinder (6)
2. Equal segments of a bisected chord (6)
3. The Crossed Ladders problem (6)
4. The Difficult Converse (4)
5. The Flying Bird problem (4)
6. The Broken Flagpole problem (4).



In 23 years, it has been presented 17 times, and as generally the enquiries have originated from non-mathematicians, through members of the Association, and thence to the bureau (which, like the N.S.W. Coal Tribunal, paradoxically consists of one person, although in dire straits he has the whole Association behind him), we have been faced with considerable calculation, for solutions have had to be simplified to avoid difficult equations, graphical solutions and the Integral calculus, which it appears the average enquirer does not want.

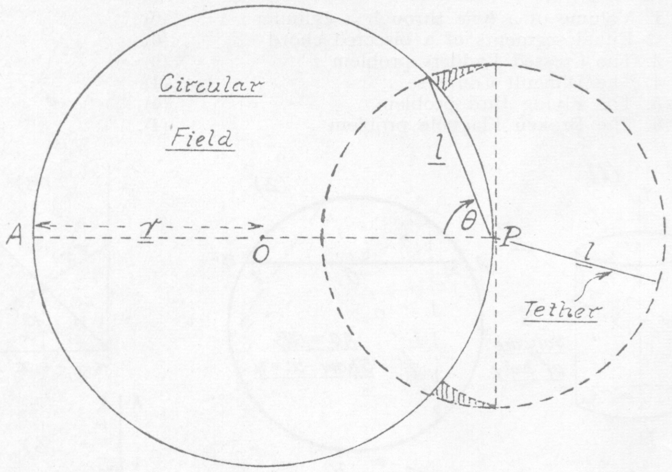
The original problem appears to have been as follows :

> A goat is tethered to a point on the circumference of a circular field of given area. What must be the length of rope for the goat to graze over exactly half of the field ?

Now this is tough enough, and having once solved it one would think that a simple transcription and a reproduction of the formula would satisfy enquirers. But this is not so ! They vary the fraction of the field from $\frac{1}{2}$ to $\frac{1}{4}$ or $\frac{2}{3}$ or other, and the area may change from the usual one acre to a circle of radius of anything from 100 yards up to half-a-mile, all requiring an accurate answer ; or again they will ask to know the grazable area with a tether of given length, or they start a new line of thought by having the goat graze *outside* the circular field (which involves an involute of a circle), with all the previous variations again repeated.

Some years ago, in exchanging experiences with Mr. G. A. Garreau, director of the Problems Bureau of *The London Mathematical Association*, on the question of dealing with these " Hardy Annuals ", he wrote :

> " Recently I programmed the ' Grazing Goat ' for our electronic computer' solving the equation by Newton's method. This was in reply to an enquirer who wanted considerable accuracy for some unknown reason."



GRAZING GOAT PROBLEM

It seems, therefore, that those with whom this problem eventually comes to rest have, to commit a " double-entendre ", about come to the end of their tether. I propose, therefore, to take a leaf out of Mr. Garreau's book (or rather a line out of his letter), where he writes : " Many of the (England) queries are answered just by sending a filed solution ", and give here two of the simpler equations, from which future enquirers, all unsuspecting, may by simple substitution and some mechanical calculations, derive the answers they seek, to an order of accuracy restricted only by their industry, in the peace and quiet of their own studies.

For the Internal area,

$$A = r^2(2\theta \cos 2\theta - \sin 2\theta + \pi)$$

For the External area,

$$A = \frac{\pi l^2}{2} + \frac{l^3}{3r}$$

where r is the radius of the circular field, l is the length of the tether, and θ is the angle whose cosine is $\frac{l}{2r}$.

The formula for the external area, when the tether is greater than the semi-perimeter of the field, needs special treatment, and is much more difficult.

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