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In Hogben's well known " Mathematics for the Million ", we read in the opening paragraph of chapter I that " Algebra was Arabic to Diderot ", and that the alleged mathematical proof by Euler of the existence of God was so far beyond Diderot's comprehension that he fled from the Russian court in humiliation. That this story is largely fictitious is now generally accepted,[1] even though various authors accepted it, including De Morgan, Bell and Cajori, who added several unsubstantiated details of their own.

Denis Diderot was in fact a mathematician of some ability. Julian Coolidge, in his recent " Mathematics of Great Amateurs " (Oxford, 1949), includes Diderot among his selected sixteen, and thus places him in a class with Omar Khayyam, Leonardo da Vinci, Albrecht Durer, John Napier, Blaise Pascal and William George Horner. Coolidge says of him that he was " a man of keen and inquiring mind, who in his earlier years was really interested in mathematics, especially in their applications ". Coolidge goes on to discuss Diderot's mathematical treatment of the acceleration of any point of a stretched elastic string undergoing small vibrations, also his memoir on the geometry of the evolute of a circle, and notes that Diderot was " an ardent advocate of Descartes' thesis that it is a great mistake in geometry to limit ourselves to what can be accomplished with the sole aid of a ruler and compass ". He also notes that Diderot, in his investigation of the motion of a pendulum, had some difficulty with the indefinite integral,

$$\int \frac{(b+x)dx}{\sqrt{(2ax-x^2)}},$$

and concludes by saying, " I cannot leave Diderot without expressing my admiration for his really stimulating mathematical work when his other interests were so large and varied ".

It is therefore not surprising that in a hitherto unpublished manuscript of Diderot, recently found by Professor Dieckmann of Harvard University, we discover an attempt to " square the circle ". Although the attempt failed, as naturally it would, his method of attack on the problem is worth our consideration, based as it is on the theorem of the " Lunes of Hippocrates ".[2] And we must excuse Diderot for spending his efforts on this time-honoured problem, for it was not until 1882 that Lindemann finally proved that $\pi$ was transcendental, and Diderot's manuscript is certainly dated earlier than 1771.[3]

Of all the problems which have persistently occupied the minds of real mathematicians and attracted the attentions of non-mathematicians throughout history, perhaps none has been so celebrated as the problem of " squaring the circle ". The very name has had a fascination for anyone with an enquiring mind. The problem proposes to draw with compasses and straight-edge alone a square equal in area to a given circle. It is referred to as the " quadrature of the circle ", and the problem of finding in a similar manner a straight line equal in length to the circumference of a given circle was soon found out to be in essence the same problem, the two finally reducing, in all essentials, to an exact determination of the value of $\pi$. It is certainly the problem to which most time and effort have been devoted, by men of all ages, nations and professions, and the literature on the subject has been enormous.

If one were seeking merely a very close approximation, then it can be said that the problem has been solved many times, and to an accuracy as close as anyone could wish. Such, however, is not the inherent difficulty of the problem. The real difficulty has been to produce a construction, with the Euclidean restrictions referred to, which shall be capable of proof by previously known ideal theorems, whose demonstration has been accomplished by the same ideal methods of the classical Euclidean geometer. These restrictions have been self-imposed, but by using other agencies and other instruments, the quadrature has been effected by many people, including the early Greeks. The age-old problem still presents a challenge which some find impossible to resist, more especially those not well equipped to cope with it, and ignorant of earlier work and literature concerning it. As long ago as 1775 the French Academy refused to examine any further solutions of the quadrature. Even today the Mathematical Association of America receives a half dozen or so solutions every year.

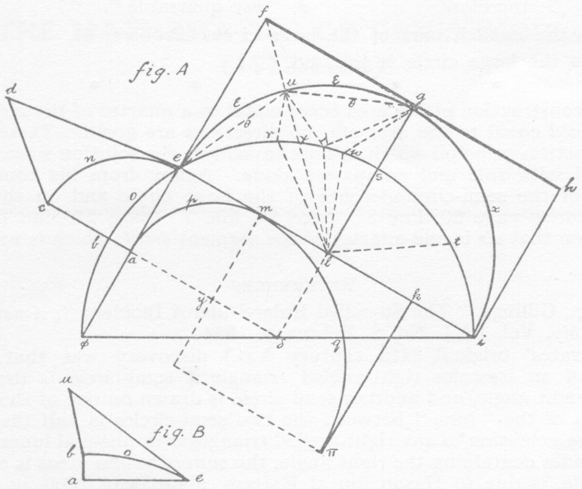
It was not until the nineteenth century that the final rigorous proof was given (Lindemann, 1882) that $\pi$ is transcendental, and since then mathematicians have known definitely that squaring the circle by classical Euclidean means is impossible. Up till that time, however, it was perfectly reasonable that mathematicians should attack the problem with the hope that they might succeed where their predecessors had failed, and we can assume that a man of Diderot's philosophic temperament and undoubted mental gifts would certainly be one to attempt it.

His attack on the problem was a sensible one. He already knew that the sum of two lunes was rectifiable; why not the difference of two lunes? He establishes in his first proposition that the difference between two lunes is one-twelfth of the area of a circle (Problem 1), a piece of work which he tells us is original with him. He then attempts to show that this difference is also expressible in terms of a rectilineal area, and if he succeeds, he has of course squared the circle. How he failed in this is shown by consideration of his construction, which is as follows:

18.

*Problem.*

*To find in terms of given rectilineal areas the value of the difference between two lunes, one of which is produced by a circle drawn on a side of an inscribed hexagon, and the other produced by a circle drawn on a side of an inscribed equilateral triangle.*




In Fig. A, let the right-angled triangle $\varphi ei$ be the triangle whose side $\varphi e$ is equal to the side of the inscribed hexagon.

Let the semi-circle $\varphi boer\psi\lambda wsti$ be drawn on $\varphi i$ as diameter of the large circle.

Let the semi-circle $e\varepsilon u\varepsilon gxi$ be drawn on $ei$ as diameter of the intermediate circle.

Let the circumference of the larger semi-circle be divided into 6 equal parts, $\varphi b$, $be$, $er$, $rs$, $st$, $ti$.

Let the circumference of the intermediate semi-circle be divided into 4 equal parts, $eu$, $ug$, $gx$, $xi$.

From the centre $\delta$ let the perpendicular $\delta b$ be drawn, dividing the line $\varphi e$ and the arc $\varphi boe$ into two equal parts.

Upon the arc *boe* construct the area *bcneo* equal to $\frac{1}{4}$ of the large lune $eu\varepsilon gxits\psi re$.

Upon the area *cne* construct the area *cned* equal to the area *efu*, or $\frac{1}{4}$ of the area $efghixg\varepsilon ue$; the arc $\psi l = \text{arc } r\psi$[4], and the sector[5] $\lambda lw = \frac{1}{4}elr$.

Let the arc $apm = \frac{1}{4}$ of the circumference of the small circle.

Let the arc $\pi qm = \frac{1}{4}$ of the circumference of intermediate circle, and let the area $abcdefghik\pi qmpa = \text{Fig. A}$.

The area $efghixg\varepsilon ue =$ half the square on the diameter of the intermediate circle, minus half the intermediate circle = Fig. K.

Let the large lune $= gl$.

Let the small lune $= pl$,

and all the rectilineal areas which may enter into the discussion which follows we will designate by the term $\pm q$.

In Fig. B, let the portion *aboe* be equal to the area *aboe* in Fig. A, and the portion *bueo* be equal to the portion $e\theta ur$ of the same Fig. A. That done, I give the solution in two parts to make it easier to follow.

The solution which Diderot gives covers eight pages of disparate written matter, which I am unable to follow step by step. In brief, however, he dissects Fig. A in three separate ways, but only in his last dissection does he make his statements clearly enough to be followed without ambiguity or uncertainty. He concludes thus:

$$-7/9\lambda lw = \pm q$$

But $\qquad \lambda lw = \frac{1}{4}elr = \frac{1}{4}d$

$$7/36d = \pm q$$

therefore $\qquad d = 36/7q$

therefore $\qquad d$, " est quarrable ".

But $d$ is the twelfth part of the large circle (Problem 1).

Therefore the large circle is squared.

* * * * *

For the construction of the area *bcneo* equal to a quarter of the large lune, and of the area *cned* equal to the area *efu*, no directions are given. These steps alone constitute a petitio principii which at once invalidate the solution since they cannot be performed with rule and compasses alone. Again, from his construction, *er* is one-sixth of the semi-circumference of the large circle and on three separate occasions, namely page 20, line 8; page 23, line 7; page 27, line 4.[6] he makes the assumption that *elr* is one-quarter of the segment *erstil*, which is wrong.

## References

[1] See, e.g., Gillings, " The So-called Euler-Diderot Incident ", *American Mathematical Monthly*, Vol. LXI, No. 2, February, 1954.

[2] Hippocrates' original (4th century A.D.) discovery was that, if on the hypotenuse of an isosceles right-angled triangle a semi-circle is drawn passing through the right angle, and another semi-circle is drawn on one of the other sides, then the area of the " lune " between the two semi-circles is half the area of the triangle. The extension to *any* right-angled triangle with unequal lunes constructed on both the sides containing the right angle, the sum of whose areas is equal to that of the triangle, is due to Hasan ibn al Haitam, about A.D. 1000, as is noted by Coolidge, p. 45, in the chapter on Leonardo da Vinci. There, he notes that the famous da Vinci " deduced numberless particular instances of rectifiable figures bounded by circular arcs ".

[3] 1771 is the date on a letter of Diderot's biographer, Nageon, which was included with the other manuscripts. There he states that M. Condorcet had seen Diderot's original construction and had discovered therein some paralogisms, which Diderot had subsequently repaired.

[4] Error in holograph for $\psi\lambda$.

[5] $\lambda lw$ is not a sector, of course.

[6] Page numbers refer to Diderot's page numbers in the manuscripts.

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