EGYPTIAN ARITHMETIC

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323. *Egyptian Arithmetic.*

MULTIPLICATION. The arithmetic of the ancient Egyptians was based upon their knowledge of the " Twice-Times " table, and their ability to find two-thirds of any number or fraction. With these two simple concepts, they developed mathematical treatments of problems in areas, volumes, in surveying, building and shipping, culminating perhaps in the construction of the Pyramids, the sole extant member of the " Seven Wonders of the Ancient World ".

If an Egyptian scribe had to multiply two numbers together, say 13 and 7, he would first choose one of the numbers (say 13) as the multiplicand, and then keep multiplying by 2, until his multipliers totalled 7, thus ...

1	13
2	26
4	52.

His mental arithmetic needed to be pretty good, especially for large multipliers, for he had to know when he had gone sufficiently far with his repeated doubling. In this simple case he would have noted that

$$1+2+4=7,$$

and he would put check marks alongside these multipliers to indicate this. Then his sum would look like ...

	\\1	13
	\\2	26
	\\4	52
Totals	7	91

Since each line has a check mark, then he must add each number in the right-hand column, and his answer is 91.

Now suppose that he chose instead 13 as the multiplier. Then his multiplication sum would look like this ...

1	7
2	14
4	28
8	56.

Again he notes that in his left-hand column of multipliers

$$1+4+8=13,$$

and he would put check marks alongside 1, 4 and 8.

	\1	7
	2	14
	\4	28
	\8	56
Totals	13	91.

Because 2 is not checked, he must be careful not to add in 14, in the right-hand column, where

$$7+28+56=91.$$

Whether the scribes knew it or not, or whether they simply took it for granted, we shall never know, but the property of numbers of the series

1, 2, 4, 8, 16, 32, ...

such that every integer can be uniquely expressed as the sum of some of them, (e.g., $45=1+4+8+32$ or, $52=4+16+32$), is the principle underlying the working of a modern electronic computer.

DIVISION. Let us suppose the scribe wished to divide 92 by 4. His technique was to multiply the divisor, in this case 4, still by simple doubling, until he reached the dividend 92. He did not say to himself, " I will divide 4 into 92 ". He said, " What must I multiply 4 by to get 92 ? " So his work looked like this ...

1	4
2	8
4	16
8	32
16	64.

At this stage he stops, for a further doubling would give 128, and this is greater than 92 so that he would have gone too far. But he still has some mental arithmetic to do, for he has to choose those numbers of the right-hand column which will add up to 92, and put check marks alongside them. He thus finds that

$$4+8+16+64=92$$

and the sum appears as ...

	1	4/
	2	8/
	4	16/
	8	32
	16	64/
Totals	23	92.

Again he must be careful to omit the 8 of the multipliers in the left-hand column, as it is opposite the unchecked 32 in the right-hand column, and so his answer appears as

$$1+2+4+16=23,$$

the required quotient.

A division sum might have an answer involving fractions, but the scribe did not change his methods. Suppose the division was $153\div6$. His working would appear as ...

1	6
2	12
4	24
8	48
16	96.

Here of course he must stop, because a further doubling would take him well past

153 to 192, and thus too far. Again his mental arithmetic comes into play and he finds that the nearest he can get to the dividend 153 is

$$6+48+96=150,$$

which is 3 short, so that he must somehow get an additional 3 in the right-hand column. And he is able to do this easily by taking one-half of 6=3, by means of his twice-times table, so that he now has ...

	1	6 /
	2	12
	4	24
	8	48 /
	16	96 /
	$\frac{1}{2}$	3 /
Totals	$25\frac{1}{2}$	153.

His quotient is obtained by adding those numbers in the left-hand column which correspond to the check marks in the right-hand column, namely

$$1+8+16+\tfrac{1}{2}=25\tfrac{1}{2},$$

and this is the answer to his division sum.

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