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Arithmetical Progression in Ancient Egypt

In a letter on Problem 64 of the Rhind Mathematical Papyrus (RMP 64), Mr. Gillings (1968) claims that ancient Egyptian mathematicians were familar with 'the equivalent of the modern algebraic formula for the sum of n terms of an Arithmetical Progression'.

This seems implausible to me. I think the following a more likely explanation of the thought behind the solution of RMP 64, an explanation, moreover, that throws light on the place of intuition in elementary mathematics.

Suppose the 10 terms of the A.P., with spaces between them, set out in order and the average term indicated by an arrow, thus:

1—2—3—4—5—6—7—8—9—10
↑
average

There are nine spaces between term 1 and term 10 and therefore nine half spaces both between term 1 and the average, and between the average and term 10. One space corresponds to the difference ($\bar{8}$ (*i.e.* 8^{-1}) in the case of RMP 64), a half-space to half the difference ($\bar{1}\bar{6}$), from which the tenth term and hence all the terms can be calculated.

Intuition is involved in identifying the average computed as, 'total divided by the number of shares', with the average computed as 'arithmetic mean of largest and smallest shares'.

If ancient Egyptian mathematicians were conversant with a rule for the sum of an A.P., this analysis of RMP 64 strongly suggests that it was in the form, 'number of terms times average of first and last term'

i.e. $s = n\,.\,\{\frac{1}{2}(a + l)\}$, a formula which is often more useful than the conventional school algebra $s = \frac{1}{2}n\{2a + (n - 1)d\}$.

J. C. W. De la Bere.

Department of Mathematics,
University of Canterbury,
Christchurch, New Zealand.

12 September 1968.

Reference

Gillings, R. J. (1968): *Aust. J. Sci.*, **31**, 47.

Reply to Mr. De la Bere's Letter

The first words of the problem state that 10 hekats of barley are divided among 10 men, and nothing could be simpler than to conclude that the average share is 1 hekat. This is clearly the scribe's first step in his solution. Mr. De la Bere asks us to suppose that the scribe envisaged an average term which is not in fact there, and shows us an arrow pointing to the middle of a space, so that he can refer to half-spaces.

Then presumably he supposes the scribe to calculate 9 half-spaces . . . '*from which the tenth term and hence all the other terms can be calculated*'. But this is not possible until the average value is known, and Mr. De la Bere supposes this average value to have been determined '*intuitively*', by finding the '*arithmetic mean of the largest and smallest shares*'. How could the scribe have done this? He does not at this stage know what these shares are, nor any other shares! This is his whole problem. '*What*', he is asked, '*is each man's share?*'

If Mr. De la Bere had suggested that the average being sought, indicated by his arrow, was half the sum of the two middle terms, or half the sum of any pair of terms equidistant from the ends, or the middle term itself if the number of terms was odd, we would have no argument to confute this, if we knew the terms of the series before we started. But in RMP 64 we do not know this, nor do we know it in a related problem from the *Kahun Papyrus* IV, 3 (see *Aust. Maths Teacher*, Vol. 23, No. 3 Nov. 1967).

It would have been very gratifying to have credited the ancient Egyptians with the concept of, $S_n = \frac{n}{2}\{a + l\}$, for the sum of an A.P., but neither Mr. De la Bere's argument, nor any of the extant papyri known to me, permit us to draw this conclusion, at this stage. One can therefore understand my earlier surprise, when the more sophisticated form, $S_n = \frac{n}{2}\{2l - (n - 1)d\}$, should have emerged from a closer examination of RMP 64. If they were aware of the simpler form '*intuitively*', we have no real evidence of it.

R. J. Gillings.

School of History and Philosophy
of Science,
University of N.S.W.,
Kensington, N.S.W., 2033.

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