MATHEMATICAL NOTES

326. THE AREA OF A CIRCLE

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Reprinted from *The Australian Mathematics Teacher,* Vol. 23, No. 1, April, 1967.

326. *The Area of a Circle.*

Standard textbooks on the History of Mathematics generally tell us that the Babylonian value for π was 3, and that the Egyptian value was the more accurate value of $\frac{256}{81}$ or $\left(\frac{16}{9}\right)^2$, and leave the matter at that. It is certain that neither of these early civilizations had any concept of π being a constant ratio for all circles,[1] namely, the ratio between the circumference and the diameter.

By examining Problem 50 of the Rhind Mathematical Papyrus (RMP), we can establish the Egyptian value for π as follows.

The problem states : Example of a round field of diameter 9 khet. What is its area ?

The length of a *khet* was 100 royal cubits, and a square khet was called a *setat,* which is approximately three-quarters of an acre.

The scribe directs ...

> Take away $\frac{1}{9}$ of the diameter, namely 1. The remainder is 8.
>
> Multiply 8 times 8, it makes 64. Therefore it contains 64 setat of land.

[1] What appears to be a simple concept, that " *all circles have the same shape, and hence are similar* ", was perhaps just as unexpected to the Babylonians and Egyptians as the concept that " *all parabolas have the same shape, and hence are similar* " is to us.

Do it thus

1	9
$\frac{1}{9}$	1

This taken away leaves 8.

Now multiply 8 by 8.

1	8
2	16
4	32
\8	64

Its area is 64 setat.

Here, as elsewhere, the scribe finds one-ninth of the diameter and subtracts it, and then squares his answer.

Put briefly, he would say : *To find the area of a circle, square* $\frac{8}{9}$ *of the diameter.*

To show that this is equivalent in modern terms to taking $\pi = \frac{256}{81}$, we write ...

$$\text{Area} = \left(\frac{8}{9}d\right)^2 \quad \text{where } d \text{ is the diameter,}$$

$$= \left(\frac{8}{9} \times 2r\right)^2 \quad \text{where } r \text{ is the radius,}$$

$$= \left(\frac{16}{9}r\right)^2$$

$$= \frac{256}{81}r^2,$$

and since $A = \pi r^2$, we see that the Egyptian equivalent of π was $\frac{256}{81}$ or $3 \cdot 1605$.

This is a reasonably close approximation to the Archimedean value of $3\frac{1}{7}$ or $3 \cdot 1429$ established more than a thousand years later, and to the more accurate value of $3 \cdot 1416$ of modern times, all values to four significant figures. In this respect the Egyptians were in advance of the Babylonians, whose value for π was sometimes taken as 3. We can compare the areas given by the various values as follows.

ORIGIN	VALUE OF π	AREA OF CIRCLE
Babylonian	3	60·75 setat
Modern	3·1416	63·6174 setat
Archimedean	$\frac{22}{7}$	63·6429 setat
Egyptian	$\frac{256}{81}$	64·0 setat

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SEAMER AND ARUNDEL STS., GLEBE, SYDNEY