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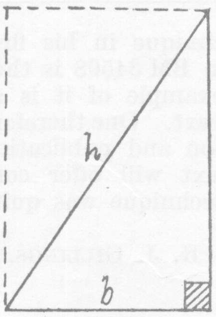
Problem No. 1 of Babylonian Cuneiform Tablet BM 34568

The Babylonian cuneiform tablet in the British Museum, BM 34568, which dates from the period of the Seleucid Kings (abt. −250), is inscribed with 19 problems which deal with the sides and areas of right triangles. The sides of these triangles are always integers, and they form what are referred to as Pythagorean Triads. When the three numbers representing the sides of a right triangle have not a common factor, they are called a primitive triad. The only triads used by the scribe in this text are

3	4	5
5	12	13
8	15	17
20	21	29

and multiples of these.



Following the scribe's practice, we will refer to the sides as length, breadth and hypotenuse (or diagonal of the rectangle).

We are to be concerned with the problems numbered 1, 2, 5 and 7. Briefly, these are

No. 1. Given $l=4$, $b=3$, find h. (Ans. 5)
No. 2. Given $l=4$, $h=5$, find b. (Ans. 3)
No. 5. Given $l=60$, $b=32$, find h. (Ans. 68)
No. 7. Given $l=60$, $b=25$, find h. (Ans. 65)

The methods of solution given by the scribe for numbers 2, 5 and 7 are quite standard, from a modern point of view, using the formula

$$l^2+b^2=h^2,$$

so that for No. 5 he gives

$$\begin{aligned}60^2+32^2&=3600+1024\\&=4624\\ \text{then } h&=\sqrt{4624}\\&=68\end{aligned}$$

and similarly for Nos. 2 and 7.

All this is very simple, and calls only for the comment that the Babylonian mathematicians were well acquainted with what we know as Pythagoras' Theorem. However, when we look at the scribe's solution for No. 1, where we would expect to see

$$\begin{aligned}4^2+3^2&=16+9\\&=25\\ \text{then } h&=\sqrt{25}\\&=5\end{aligned}$$

we find something altogether different.

The scribe directs as follows:

'You may either

(*a*) Add half of your length to your breadth, so that

$\frac{1}{2}$ of $4+3=2+3=5$,

the diagonal, or

(*b*) Add a third of your breadth to your length, so that

$\frac{1}{3}$ of $3+4=1+4=5$,

the diagonal.'

One naturally asks, is this a mere chance relationship of the numbers involved, or does it always hold? The answer is, yes it does always hold for all right triangles of this particular shape, as for example (6, 8, 10), (9, 12, 15), (12, 16, 20) . . . etc., etc., as one sees by adding a half of 8 to 6, giving 10, or by adding a third of 6 to 8, giving again 10, and the same applies to every other triad which is a multiple of the primitive triad, 3, 4, 5.

We must now ask, what of all the other primitive Pythagorean triads (of which there are an infinite number), and does there exist a simple rule for each of these? We shall find that such a rule does exist, and that it is reasonable to assume that the Babylonians were quite familiar with it.

It is now generally accepted by Assyriologists, after close examination of Plimpton 322 and other Cuneiform texts, that the Babylonians determined primitive Pythagorean triads, by evaluating

$$l=(p^2-q^2),\ b=2pq,\ h=(p^2+q^2),$$

for the length, breadth and diagonal of a rectangle, where p and q are relatively prime, not simultaneously odd, and p is greater than q.

Now if one chooses the pairs of values for p and q as under,

p	q
2	1
3	2
4	1
5	2

the four primitive Pythagorean triads used by the scribe who prepared the tablet BM 34568 are obtained as

l	b	h
3	4	5
5	12	13
15	8	17
21	20	29

The values of p and q being known (to the scribe), modern algebra shows us that

(a) $(q/p \text{ of } b)+l=h$, and

(b) $\left(\dfrac{p-q \text{ of } l}{p+q}\right)+b=h$, for

$$(a)\ (q/p\times 2pq)+p^2-q^2 = 2q^2+p^2-q^2, \\ = p^2+q^2, \\ = h, \text{ and}$$

$$(b)\ \frac{(p-q)}{(p+q)}\times(p^2-q^2)+2pq = (p-q)^2+2pq, \\ = p^2-2pq+q^2+2pq, \\ = p^2+q^2, \\ = h.$$

It is thus mathematically clear that the scribe's method for finding the hypotenuse given the other two sides of the right triangle, which he indicates in his first problem, is universally applicable, provided that the relevant values of the parameters p and q are known. Following the usual Babylonian practice of compiling tables for ready reference, we can set down what might have been the scribe's first few entries on his clay tablets, for recording the fractions of l and b, which must be added to b and l respectively to obtain the values of h.

In this table we have ignored the restrictions on p and q that they should be relatively prime and not both odd, for we are not restricted to primitive Pythagorean triads, and we preserve the detail of the arithmetical series, shown in the all important last two columns.

TABLE 1

p	q	l (p^2-q^2)	b $2pq$	h (p^2+q^2)	Fraction of b which added to l gives h	Fraction of l which added to b gives h
2	1	3	4	5	1/2	1/3
3	1	8	6	10	1/3	2/4
4	1	15	8	17	1/4	3/5
5	1	24	10	26	1/5	4/6
6	1	35	12	37	1/6	5/7
7	1	48	14	50	1/7	6/8
3	2	5	12	13	2/3	1/5
4	2	12	16	20	2/4	2/6
5	2	21	20	29	2/5	3/7
6	2	32	24	40	2/6	4/8
7	2	45	28	53	2/7	5/9
8	2	60	32	68	2/8	6/10
4	3	7	24	25	3/4	1/7
5	3	16	30	34	3/5	2/8
6	3	27	36	45	3/6	3/9
7	3	40	42	58	3/7	4/10
8	3	55	48	73	3/8	5/11
9	3	72	54	90	3/9	6/12
etc.		etc.			etc.	etc.

That the scribe used this technique in his first problem on Pythagorean triads in BM 34568 is thus well attested. But no similar example of it is at present known to me in any other text. One therefore hopes that some future translation and publication of a cuneiform mathematical text will offer confirmatory evidence that such a technique was quite generally known and practised.

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